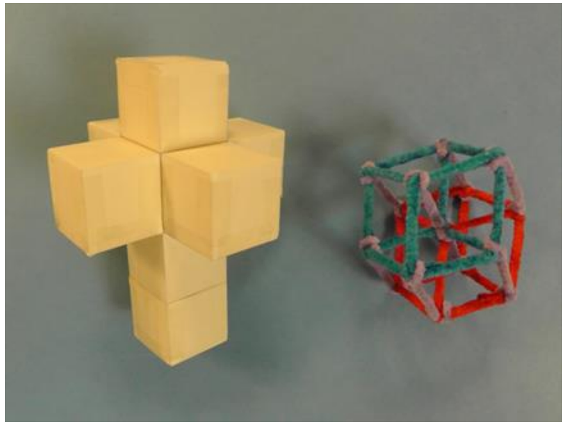
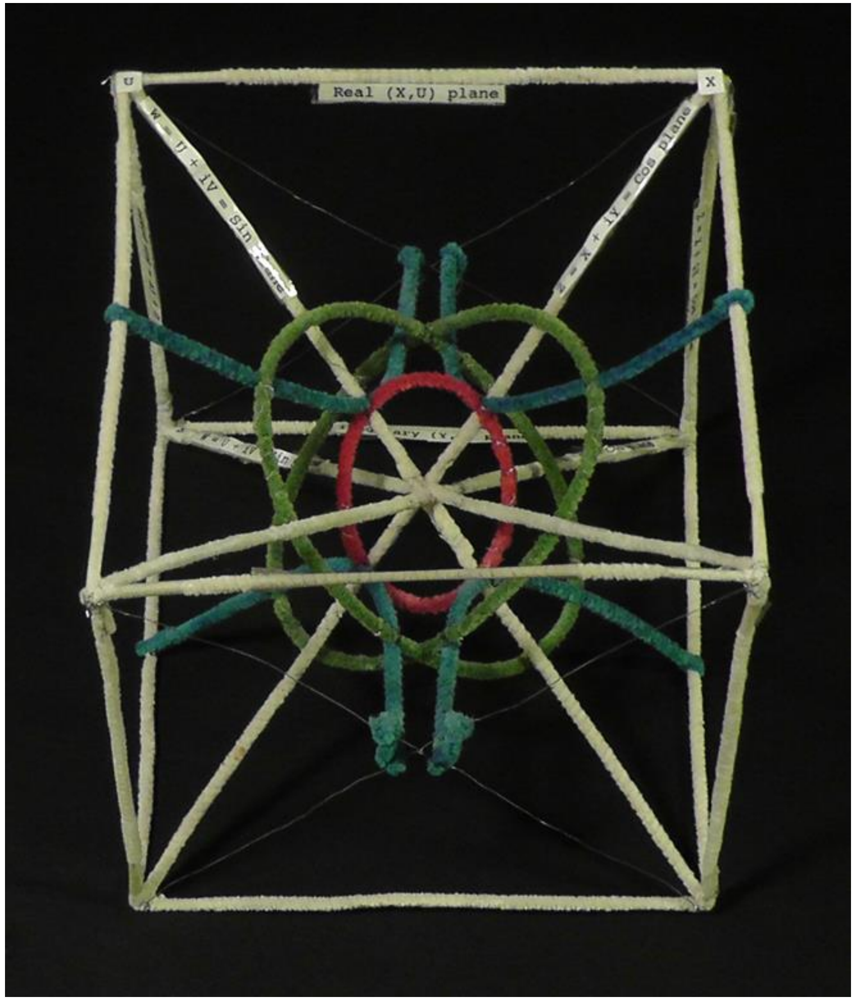
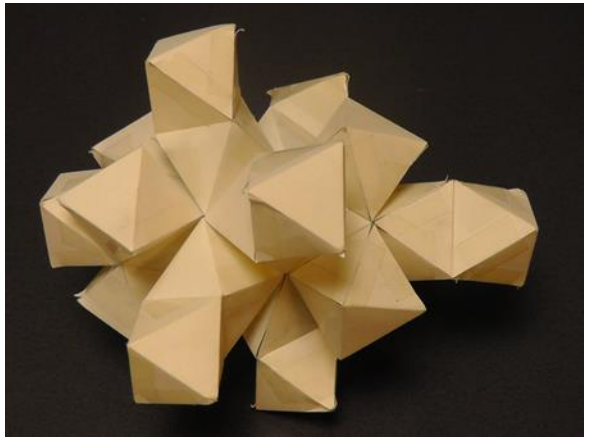
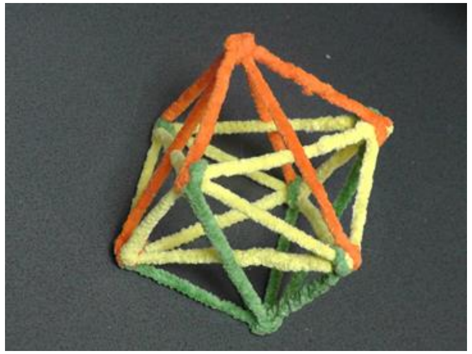
Higher Dimensional Spaces

By James Zongker









AGENDA

Why this approach?

Characteristics of Space

Regular Shapes & Lattices

Applications (highlight these):

- Mobius
- Complex Exponential Function & Trigonometry = Pretty Pictures
- Fractals
- The Easy Solution
- Differential Equations = Geometry
- More Complex Variable & Integration (Pretty Pictures)

Questions

Understanding Existence

The existence we know seems to consist of particular things that exist and interact in space through time. Ultimate understanding requires we understand space, time, and the particulars and principles driving existence and interaction.

Space is one of these fundamentals.

What is special about 3D?

Do higher dimensions allow more or less structure & regular patterns?

Understanding Basic Existence:
What is Special About Spatial Thinking?

There are different ways of perceiving, learning and intelligence/mental skills:

Visual, auditory, touching, language, math, music, body coordination, emotional, moral, etc…

You think about objective knowledge things in concepts.

Concepts can be encoded in words.

Words can summarize and communicate concepts.

You can’t actually think in words. You have to grasp & use concepts.

it can be hard to avoid mental talk echoing what you are conceiving.

This can limit mental flexibility if you lack words for it.

Spatial visualization and reasoning is DIRECT:

The mental picture is a pure concept of the object.

The symbolic representation is often a picture of the object.

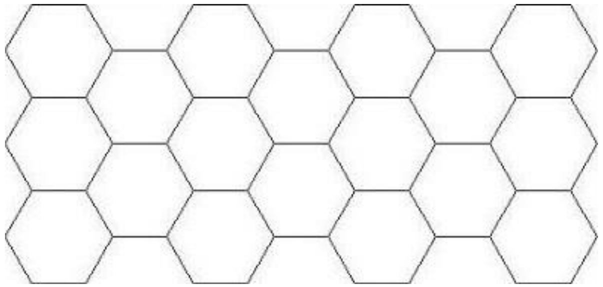
Limited by how well you can “visualize” or “mentally define” it.

What kinds of spaces are there?

<u>Space</u>

- Can have n dimensions = # of independent directions.
- Can contain stuff:

  Like a uniform lattice of identical clumps (shapes or objects).



Understanding dimensions, shapes, symmetry and uniform lattices provides a fundamental map of Euclidean spaces (like simple 3D space.)

Euclidean Spaces: Simple, continuous, flat (not curved) spaces.

How is 3D Special

<u>You need at least 3D for:</u>

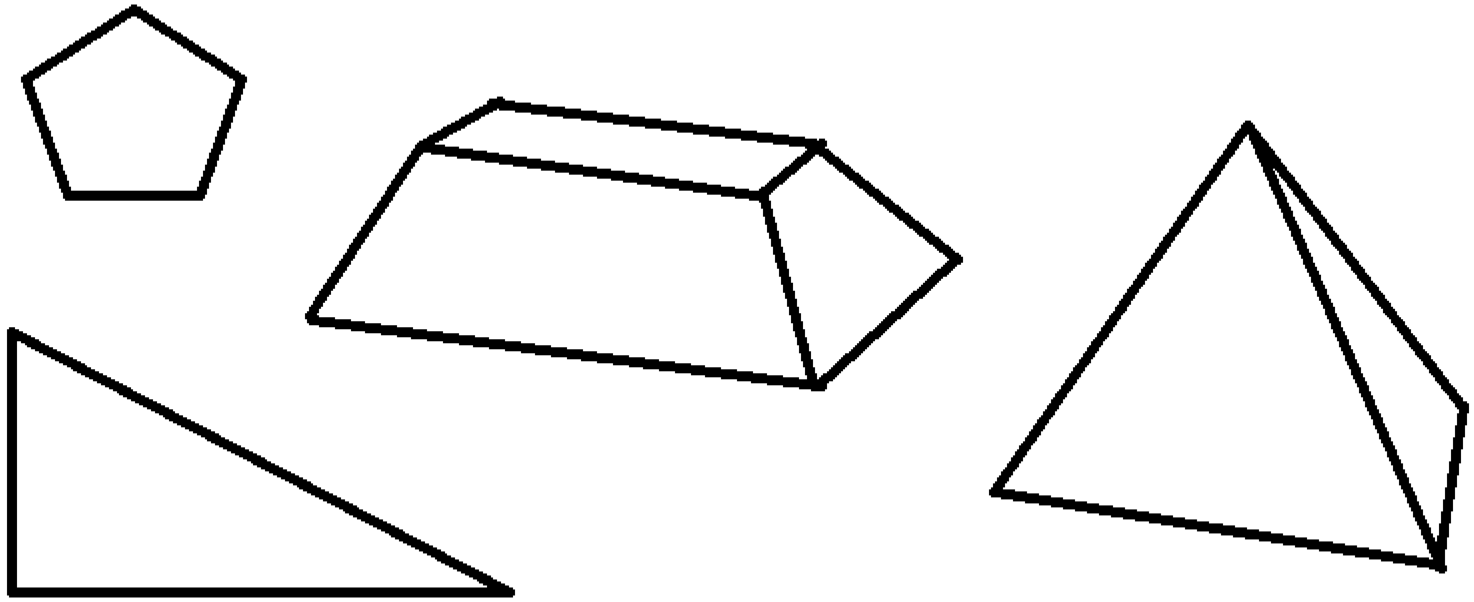
An organism to not have to eat and defecate at the same orifice (or the digestive track would cut it in half.)

Lots of lung surface area in a small volume with efficient blood flow. (interlocking fractal geometries.)

Gravity & Electricity to decrease with distance squared to allow more functional separation between all things. (Also, Magnetism won’t work same way in 2D or 4D, and light could not have a magnetic wave in 2D.)

Polytopes

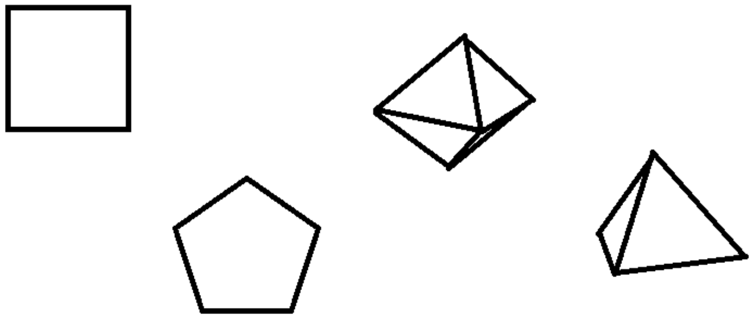
A **Polytope** is the generalization of the concept of 2D polygons and 3D polyhedrons for all n-dimensional spaces.



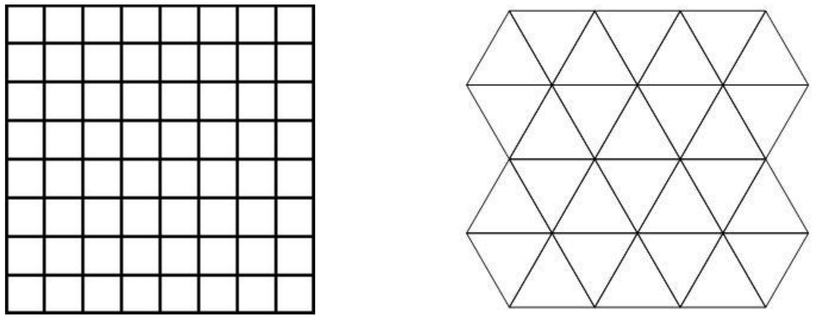
Regular Polytopes

All features (sides, edges, angles)

are alike:



A uniform lattice may be built of regular shapes, like balls or cubes stacked in the same pattern across the space.

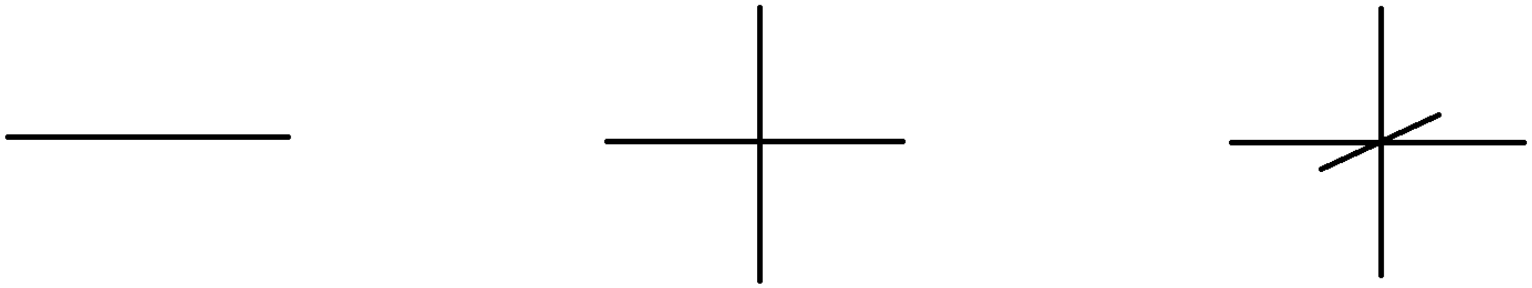


Let’s first understand regular polytopes.

Dimensionality

Pure concept:

N-Dimensional Space requires n independent directions (vectors) to span it.

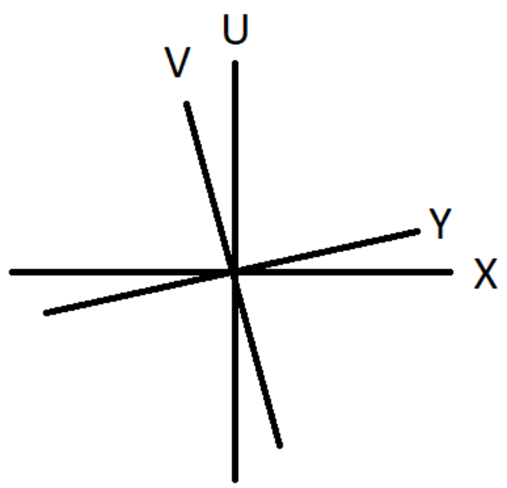


1D 2D 3D

Each of these pictorial representations motivates an idea of a space.

The 4th Spatial Dimension

Adding a 4th "spatial" (not time)
direction is easy to diagram.



What is it? Where does it go? How is it possible?

You live in a 3D existence. 4D space will not fit in 3D space.

Your mind is not evolved nor acclimated to 4D.

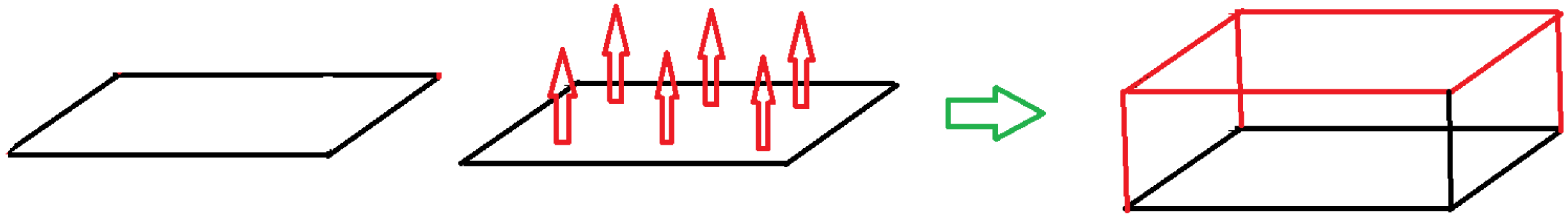
However, the pure concept of 4D (and higher) can be clearly defined.

Higher dimensional space has conceptual meaning and implications.

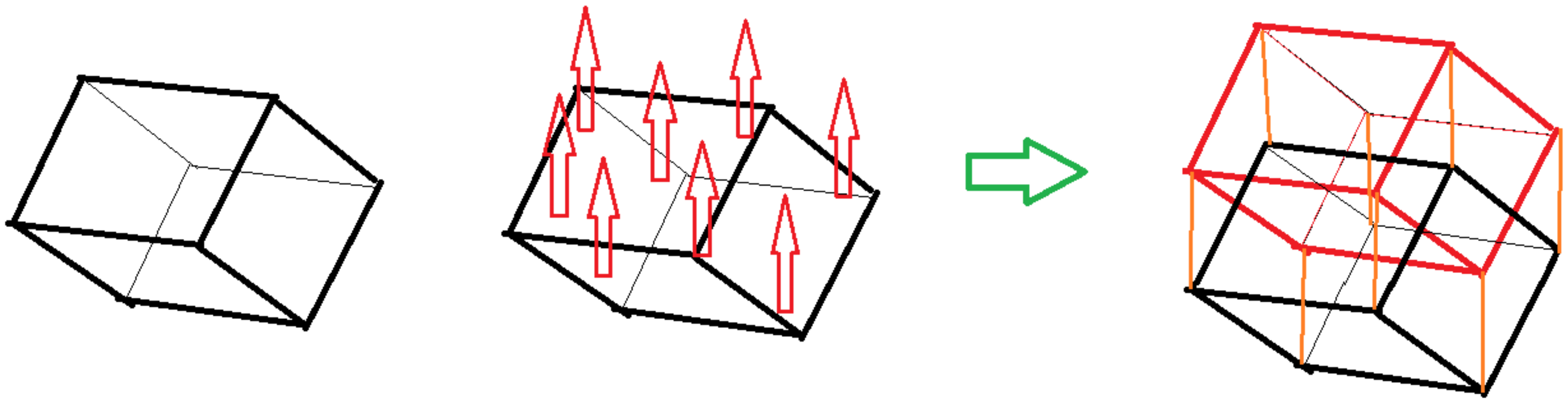
The challenge is how to conceive or visualize it to reason about it.

What is It?

The 3rd dimension is another direction imposed everyone on 2D Space

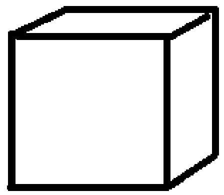


A 4th dimension is just another direction imposed at every point in 3D space.

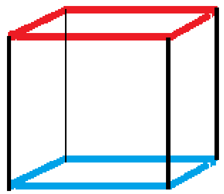


The Cube as 2 Squares

This “flat” picture looks 3D because you are used to processing it as 3D:

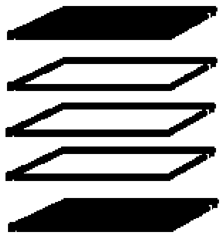


A cube can also be viewed as connecting a top & bottom square:



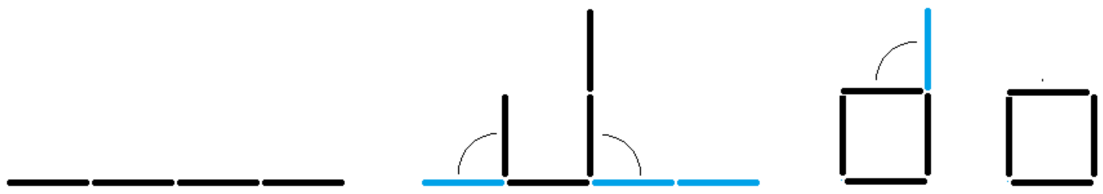
You could show it to a 2D being by passing a cube through their 2D plane:

- See nothing
- See solid square (bottom)
- See hollow square
- See solid square (top)
- See nothing

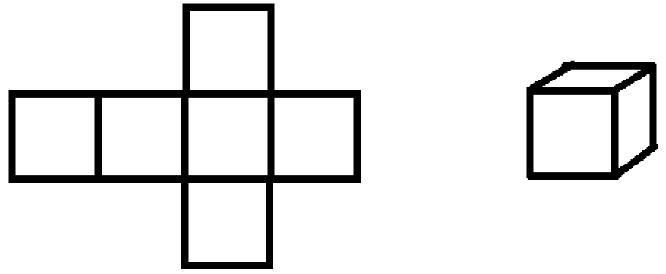


Flat Pattern Folding a Cube

Or you could explain just as you fold a square from 4 lines:



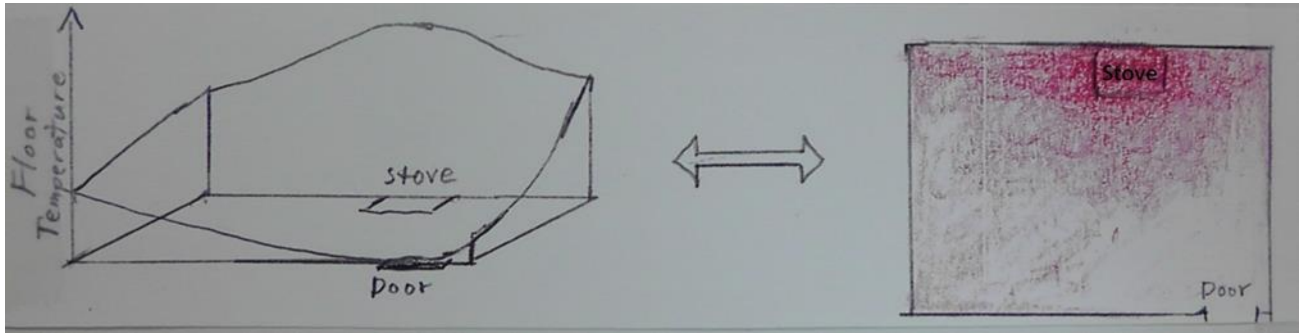
You can fold up this to make the cube:



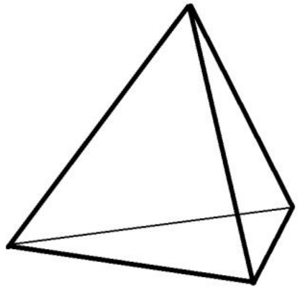
Folding that last picture gets a bit complicated for a 2D being.

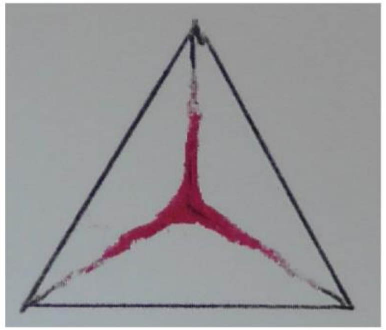
Qualitative Dimensions

You can use temperature, color or height of a surface for the same thing:



This works to illustrate a regular 3D tetrahedron in 2D:

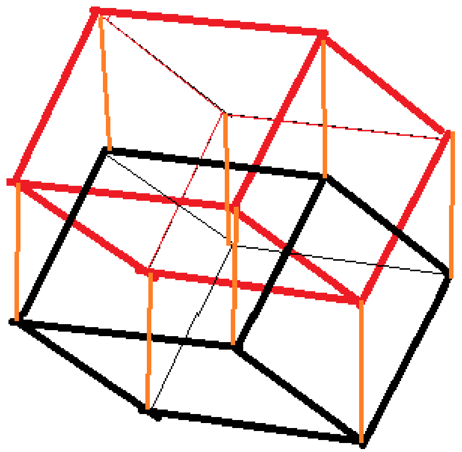




3D object 2D drawing with color = height.

The HyperCube

A hypercube (4D Cube) is a 3D cube bottom connected to a 3D cube top displaced into a 4th dimension, illustrated here with color.

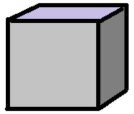


Each square side of the bottom cube is connected to the corresponding square on the top cube by a cubical side.

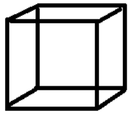
3D Slicing the HyperCube

You can also push the hypercube through 3D space much like a cube through 2D space, and see:

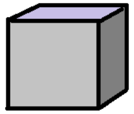
Nothing



Solid Bottom Cube



Hollow Cube (as traverse the connecting sides)

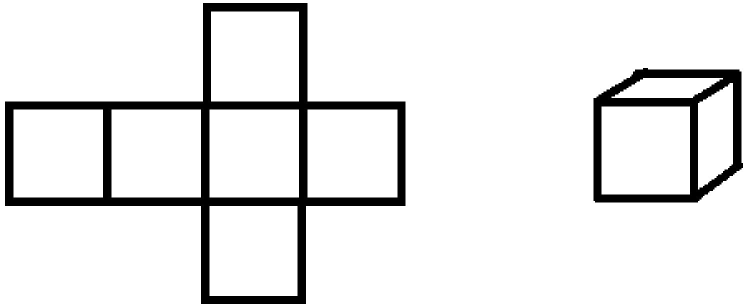


Solid Top Cube

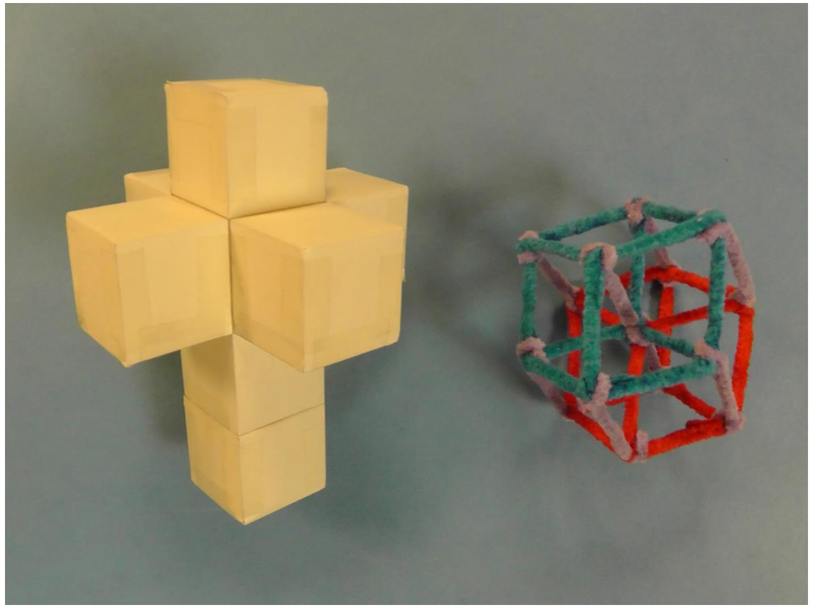
Nothing

Folding the HyerCube

Just as you can fold up a cube from its 2D flat pattern,



you can fold up a hypercube from its 3D flat pattern.



SubSpaces

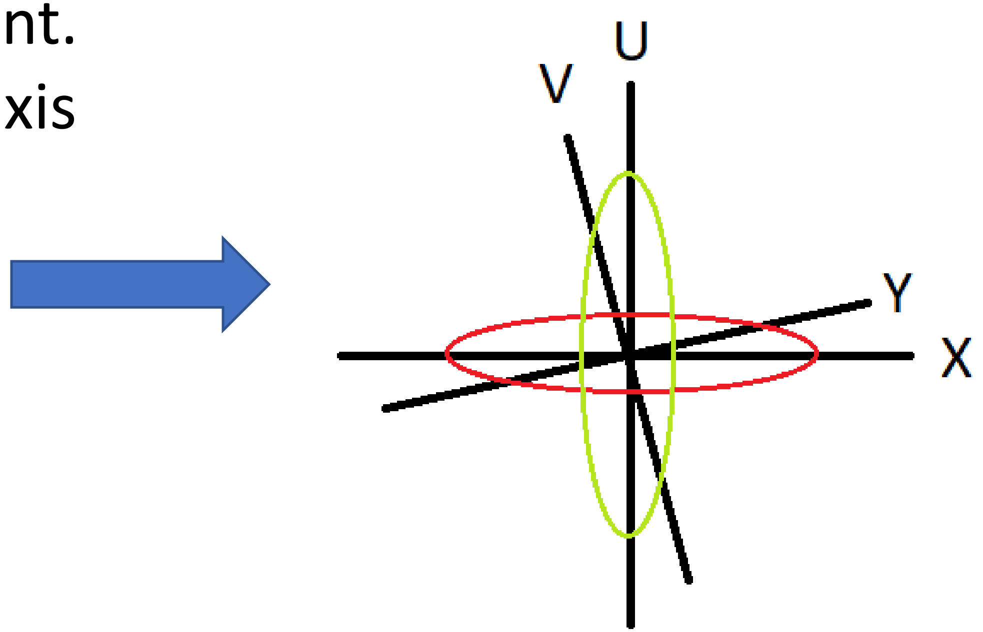
Think of these conceptually (by definition), not just visually
(as I illustrate in 3D?)

- It takes n independent directions to define or span n-D space.
- If collapse n-D space along ANY direction (even diagonal) it loses a dimension (to become an n-1 D subspace).
- An infinite n-1 subspace slices an n-D space into two regions, and can be used as an n-1 “plane” to judge if things are symmetric (mirror images) on each side of that “plane”.
- Collapsing n-D space a 2nd time along a DIFFERENT direction creates another n-1 subspace that meets the 1st collapse subspace along an n-2 D intersection subspace. The intersection is at an angle which can be measured in a simple 2D plane! (Explain and illustrate if asked)
- An n-2 D subspace can serve as the axis of a rotation of objects in n-D space. The rotation itself only exchanges 2 dimensions while the others are stationary. Illustrate with 3D model.

n-Dimensional Freedom of Motion

In addition to position, velocity and acceleration in n directions:

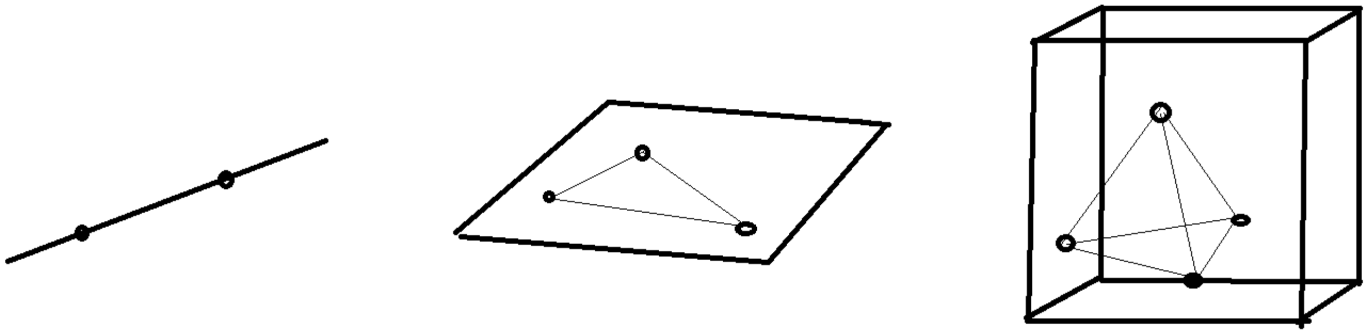
- 1D allows no rotations, only movement.
- 2D & 3D rotation can only have one axis at a given instant.
- 4D allows 2 independent rotations of a body at the same time!
- 6D allows 3 independent rotations.
- 2nD allows n independent rotations.



That could be a lot to keep track of if we moved in a world like that!

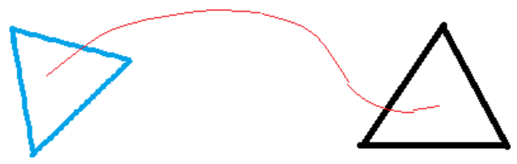
Points Defining/Spanning a Space

It takes 2 points to define a 1D line, 3 a 2D plane, and 4 a 3D space.
It takes n+1 points to define an n-D space.

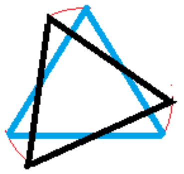


NOTE:
3 points can’t be in the same line,
4 not in same plane,
and n+1 can’t be in less than an n-D subspace.

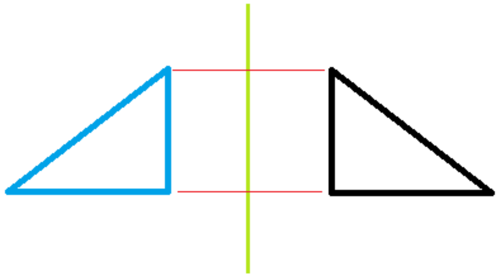
Rigid Continuous Displacement (RCD)



An RCD is a way to move n-D objects within n-D space in a continuous motion that does not distort or alter the shape, size, angles, etc of the object. (Just Moves It!)

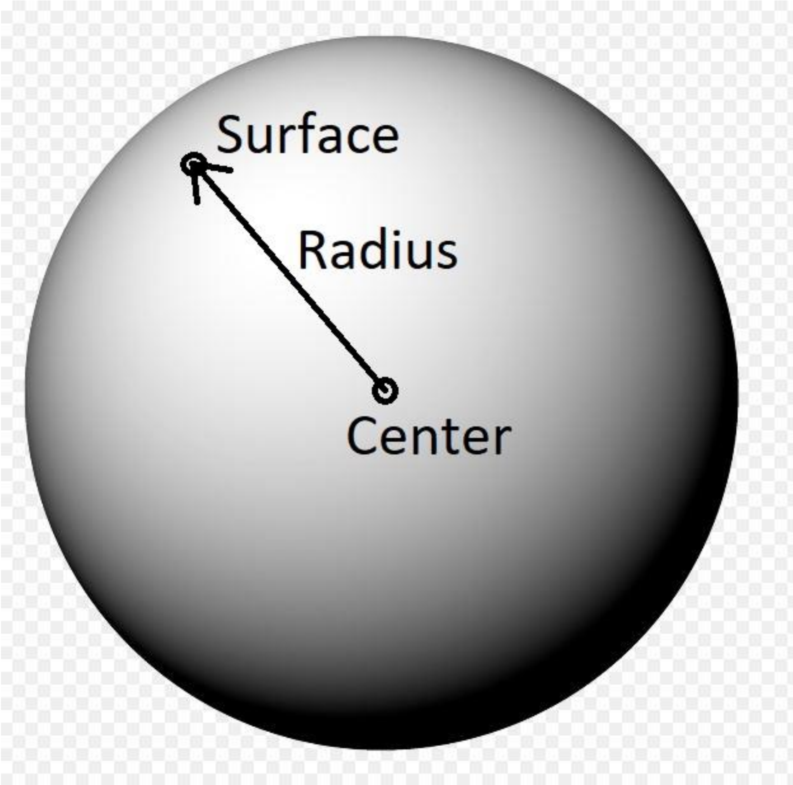


Rotations are RCDs.



Reflections across subspaces are NOT RCDs.

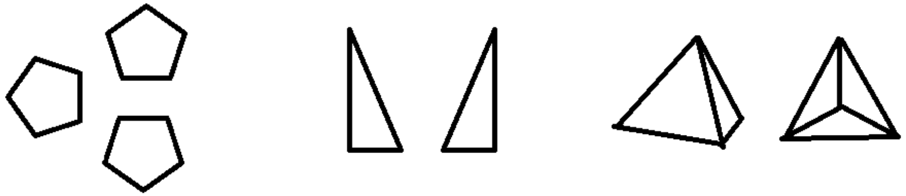
N-Sphere



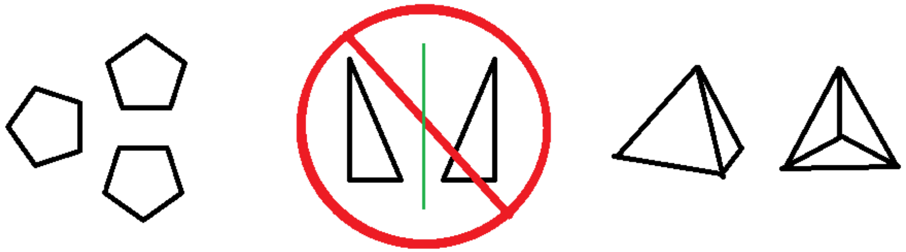
An n-sphere is all points in n-D space at some equal radius distance from one point called the center. It is a curved n-1 dimensional surface that divides n-D space into what is inside vs outside.

RCD Congruent

Two geometric shapes are **congruent** if they are identical except for where they are placed or how they are oriented.



They are **RCD congruent** (in that dimension) if one can be mapped to the other with an RCD. (Mirror images are NOT RCD congruent. HOWEVER, a higher dimensional RCD may fix that!)



Define Regular Polytope

<u>An n-D Regular Polytope (RP) is a constellation of vertex points such:</u>

- There is a positive whole number “v” of vertex points.
- The vertex points are arranged such they will not fit in less than n-D space.
- All vertex points are located on the surface of an n-sphere about its center.
- The constellation is “Regular”, meaning: If the center is fixed (cannot move), and enough other vertexes A1, A2, A3…) are selected that suffice to define where the rest of the constellation pattern must be, then if any other RCD Congruent set of vertices (B1, B2, B3, etc, which can include duplicates of the first set) is chosen, there exists a RCD that can move the entire RP such it falls on top of the original RP position, with all original vertex locations occupied by vertexes in the same locations after the RCD.

illustrate this with a 3D model as talk.

Features of Regular Polytopes

Vertexes (Vertices) = 0 dimensional points where features join up.

Adjacent Vertices = Pair of distinct vertices that at the minimum separation.

Lines = 1 dimensional line segments between adjacent vertices.

Sides = n-1 dimensional faces that divide interior from outside

Edges = n-2 dimensional features where sides meet.

Corners (not necessarily vertices) = n-3 dimensional intersection features.

Illustrate all this with 3D model as talk

RP Sides are n-1 D Regular Polytopes

<u>Think from conceptual definitions…</u> Illustrate with 3D models as talk.

For an arbitrary RP in n-D , consider a collection of some of its vertex points sufficient to define an n-1 subspace located as far as possible from the center. This n-1 subspace must intersect the n-sphere along an n-2 subspace roughly analogous to the circle where a plane slices a sphere. Therefore the points outlining this “side” lie on an n-2 sphere, and must be regular with respect to that sphere (or violate regular symmetry of the RP). Therefore the sides of all regular polytopes are n-1 D polytopes.

They must also all be of the same size (to not violate symmetry).

Folding Algorithm

All n-D Regular Polytopes are made from n-1 D Regular Polytope sides

Sides Intersect at n-2 D Edges

Sides end at n-3 D Corners where at least 3 sides meet.

Same # of sides must always meet at every corner (by symmetry)

All n-D Regular Polytopes are n-1 D flat pattern fold ups.

Illustrate all/partly if can with 3d Models as talk

Grand Simplification

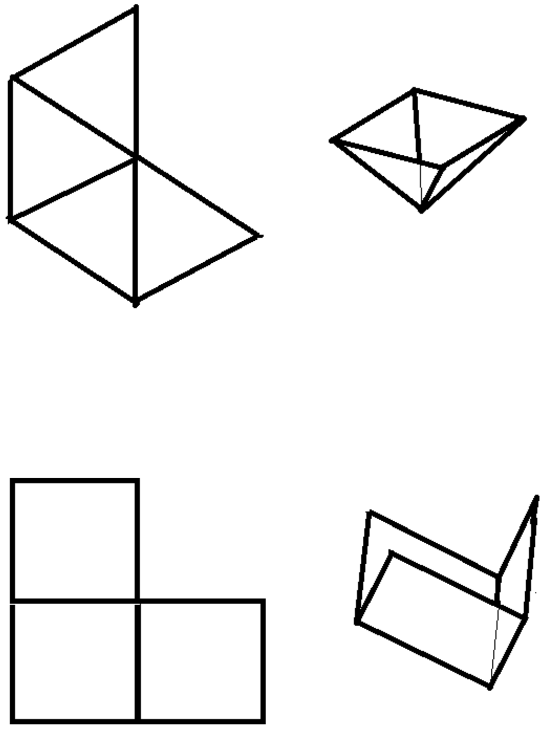
n>3 adds geometric complexity. Fortunately the basic relation of angles is fully described by just 3 dimensions:

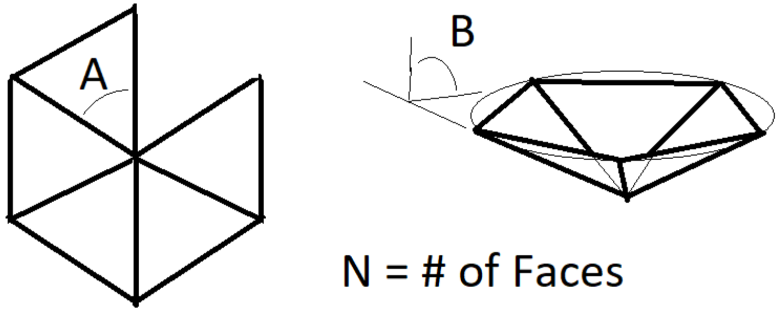
2D plane containing angles of the flat faces which are all in the same 2D plane prior to folding (all the extra dimensions just extend out of that 2D plane.)

1D added to fold the face angles up to converge together. All these folds rotate around axes consisting of all excess dimensions (beyond 3D), such all the folding 2D rotations at any one corner lie entirely within just 3 dimensions.

Universal 3D Solution

Thus, 2D and 3D figures like this suffice to diagram and compute angles.





The Folding Formula

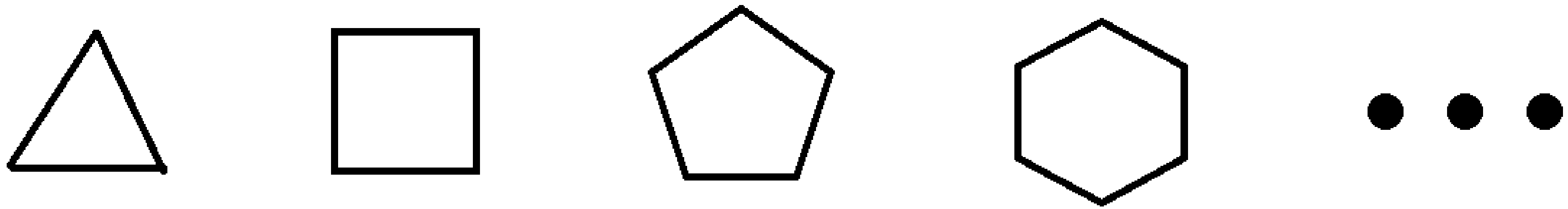
Doing the analytic geometry and trig gives this “ugly” formula:

$$B = 180^\circ - 2\,\mathrm{Tan}^{-1}\left\{ \tan\frac{180^\circ}{N}\ \sin\left(\cos^{-1}\left[\frac{\sqrt{1-\cos A}}{\sqrt{1-\cos\frac{360^\circ}{N}}} \right] \right) \right\}$$

The fold angle (B) for any RP in any dimension can be found by knowing how many (N) RPs of what angle (A) exist in the next lower dimension.

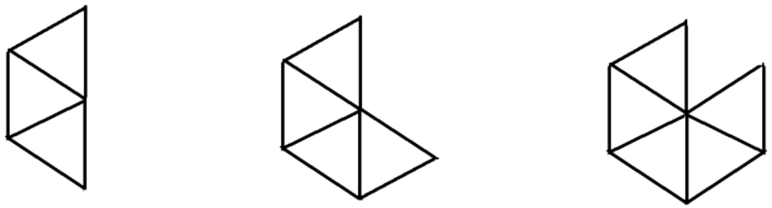
All the 2D Regular Polytopes

There are an infinite # of regular polytopes (polygons) in 2D:



3D Regular Polytopes (Triangular Sides)

3, 4 or 5 60° equilateral triangles
can be placed around a corner

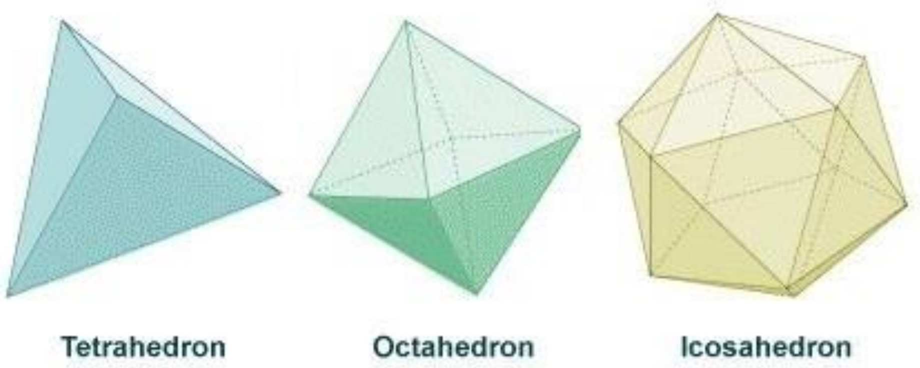


Giving folding angles of

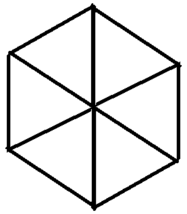
70.53° for the Regular Tetrahedron

109.47° for the Regular Octahedron

138.19° for the Regular Icosahedron

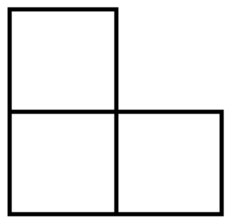


6 60° equilateral triangles total 360° such folding is impossible.

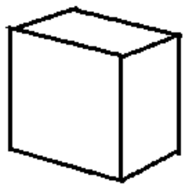


3D Regular Polytopes (Square Sides)

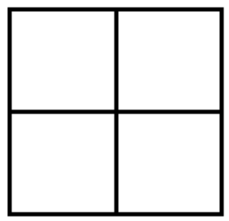
3 90° squares can be placed around a corner



Giving a folding angle of 90° to make a cube.

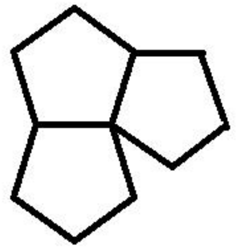


4 90° squares total 360° such folding is impossible.



3D Regular Polytopes (Pentagonal Sides)

3 108° pentagons can be placed around a corner.

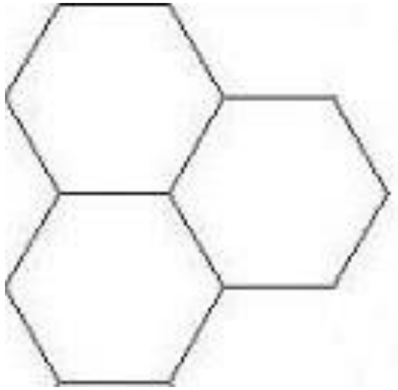


Giving a folding angle of 116.57° to make a Dodecahedron

4 108° pentagons exceed 360° such arranging and folding is impossible.

No More Regular 3D Polytopes

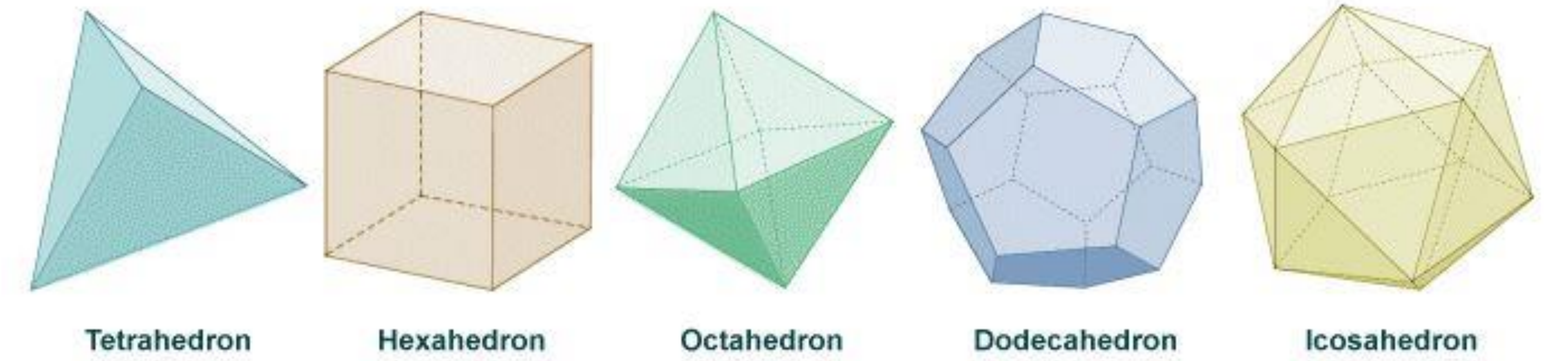
Regular hexagons have a 120° angle, such 3 of them already total 360°.



Regular polygons with 7 or more sides have angles exceeding 120° such it is impossible to arrange 3 or more about a corner for folding.

Platonic Solids

Therefore the formula correctly predicts there are only 5 RPs in 3D, which are the well known Platonic Solids.

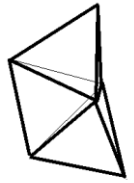


T3	C3	O3	D3	I3

4D Regular Polytopes (Tetrahedral Sides)

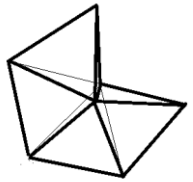
The Regular Tetrahedron (T3) has an angle of 70.53°, enabling 3, 4 or 5 of them to be arranged around an n-3 dimensional corner. (n-3 happens to be 1, or a line in the case of the 4th dimension.) This gives angles of

75.52° for 3 sides meeting:



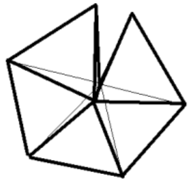
T4: 4D Tetrahedron analogue (5 sides and 5 vertexes)

120° for 4 sides meeting:



O4: 4D Octahedron analogue (16 sides and 8 vertexes)

164.48° for 5 sides meeting:

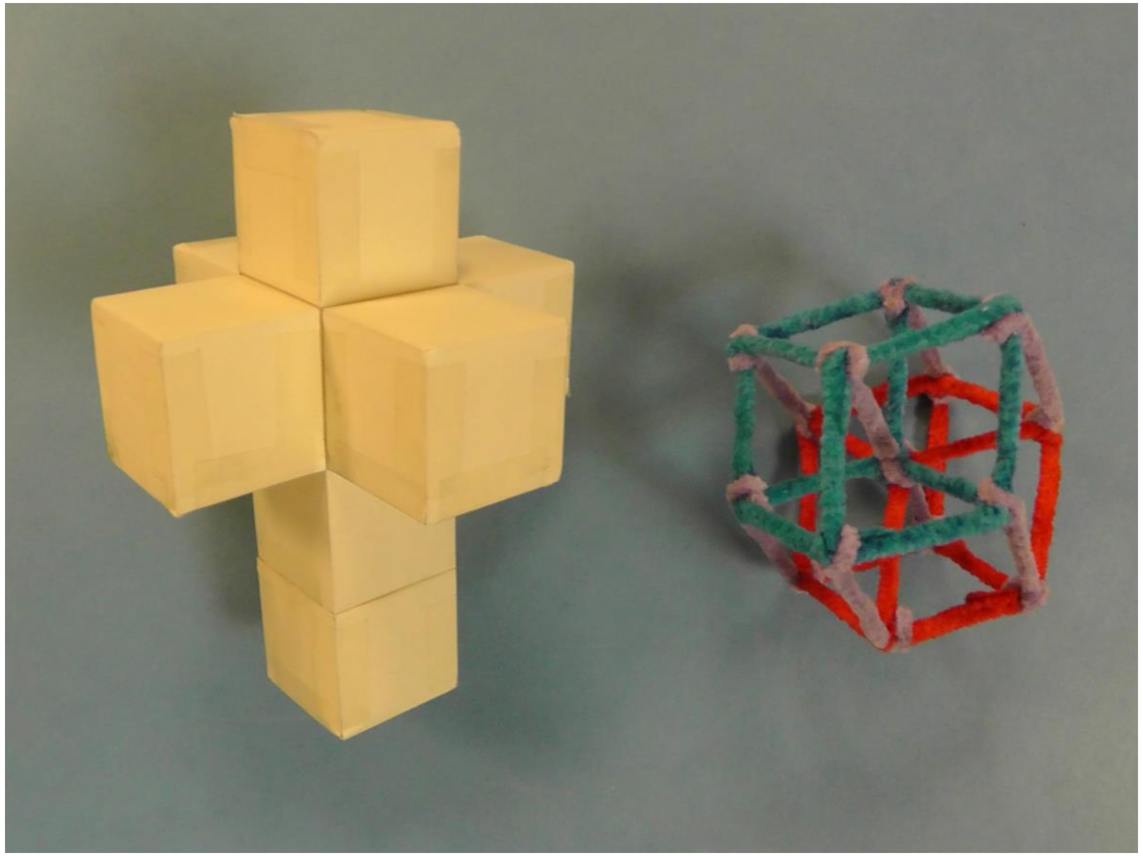


I4: 4D Icosahedron analogue (600 sides and 120 vertexes)

4D Regular Polytopes (Cubical Sides)

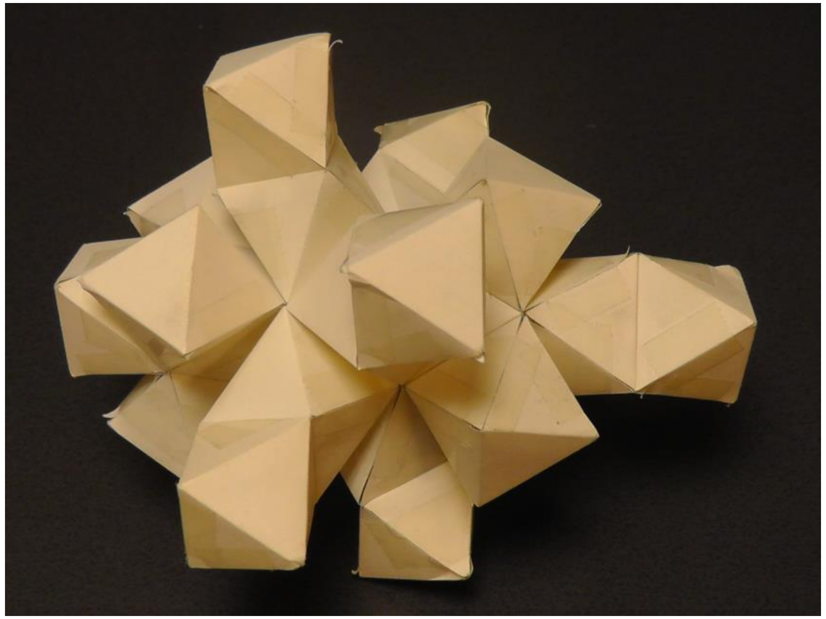
Not surprisingly, as with the 3D case, 3 90° cubes can fold to a 90° angle to give the hypercube.

However, 4 90° cubes already total 360° so no fold angle is possible.



4D Regular Polytopes (Octahedral Sides)

The Regular Octahedron (O3) has an angle of 109.47°, enabling 3 of them to be arranged around a corner and folded to a very special angle of 120°.



The flat pattern development shows it has 24 sides, with 6 octahedra arranged in a line that turns into a regular hexagon when folded up.

The adjacent vertexes are spaced the same distance apart as the 4-sphere radius, so the vertexes are the same distance from the center as they are from adjacent vertexes.

4D Regular Polytopes (Dodecahedral Sides)

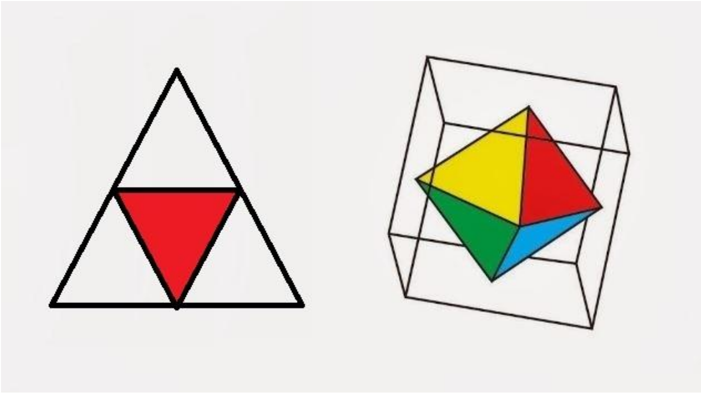
The Dodecahedral angle was only 116.57°, so 3 of them total less than 360° and can be folded to a 144° angle to give an analogue to the Dodecahedron.

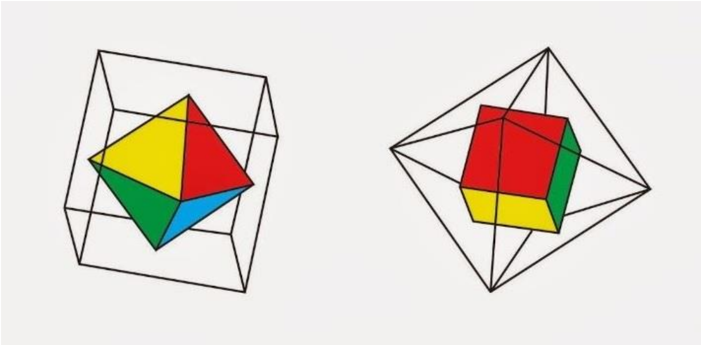
What does it look like!!!!   Way too complicated to draw it.

There is an easier way to figure out more about it…

Every RP Has a Dual

The centers of the sides of any RP define a set of vertexes for a dual RP that can be inscribed inside the original RP. This has to be true by Symmetry.



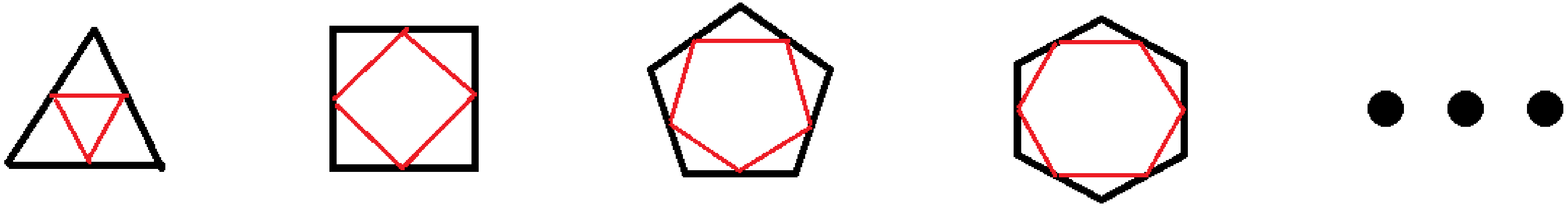


Further, the dual of a dual is a smaller copy of the original RP, so duals always occur in pairs.

Lastly, the number of sides and vertexes always swaps for its dual.

2D Duals

The dual of every regular polygon is a smaller version of itself:

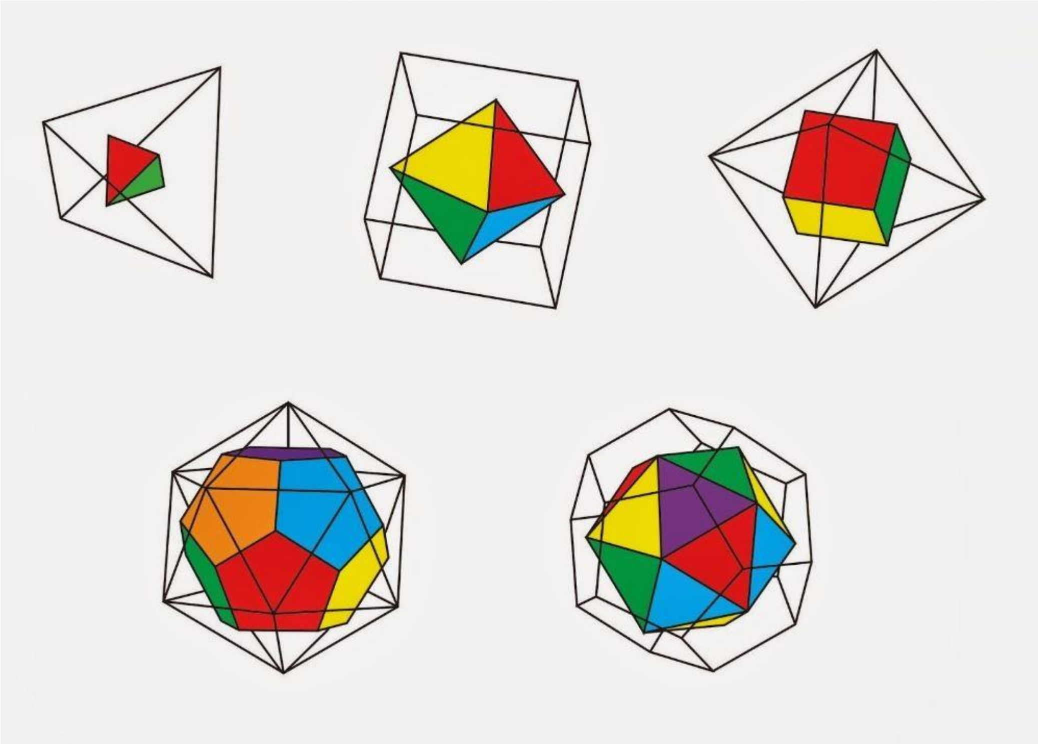


3D Duals

Tetrahedron <-> Tetrahedron

Cube <-> Octahedron

Icosahedron <-> Dodecahedron



4D Duals

T4 <-> T4 Tetrahedron analogue

C4 <-> O4 Cube/Octahedron analogue

H4 <-> H4 Hexagon analogue (has no 3D analogue)

I4 <-> D4 Icosahedron/Dodecahedron analogue

Since careful flat pattern work determined I4 had 600 sides and 120 vertexes, D4 must have 120 sides and 600 vertexes.

5D, T4 Sides

T4 had and angle of 75.52°, thus 3 or 4 may be arranged around an n-3 D (2D polygon) corner, giving

Tetrahedron analogue (T5) with 78.46° angle.

Octahedron analogue (O5) with 126.87° angle.

However it is not possible to place 5 or more T4 around a corner above 4D, because it can be shown the folding angles keep getting bigger.

Will there be an upper dimensional limit for the octahedral and tetrahedral analogues?

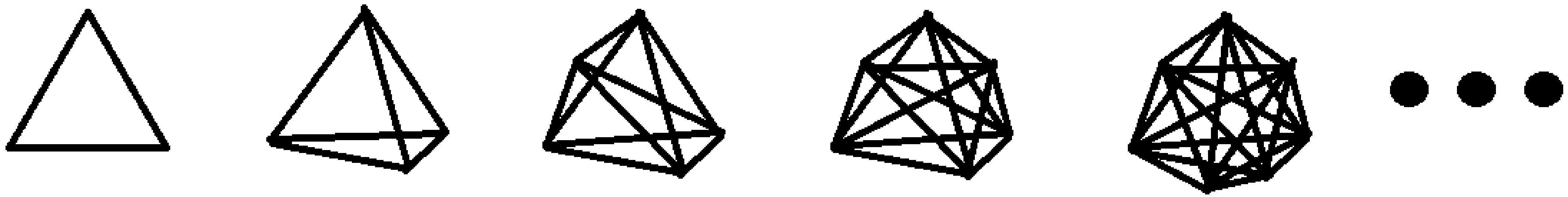
5D, Cubical Sides

By induction, for each higher dimension, it will always be possible to place 3 90° cubical sides about an n-3 dimensional corner and fold to 90°. Therefore there are analogues to the cube in every higher dimension.

Not surprising as the n-cube is the brick we'd expect to stack in a lattice that tiles n-dimensional space. That is how integer Cartesian coordinates are defined for n-dimensional space.

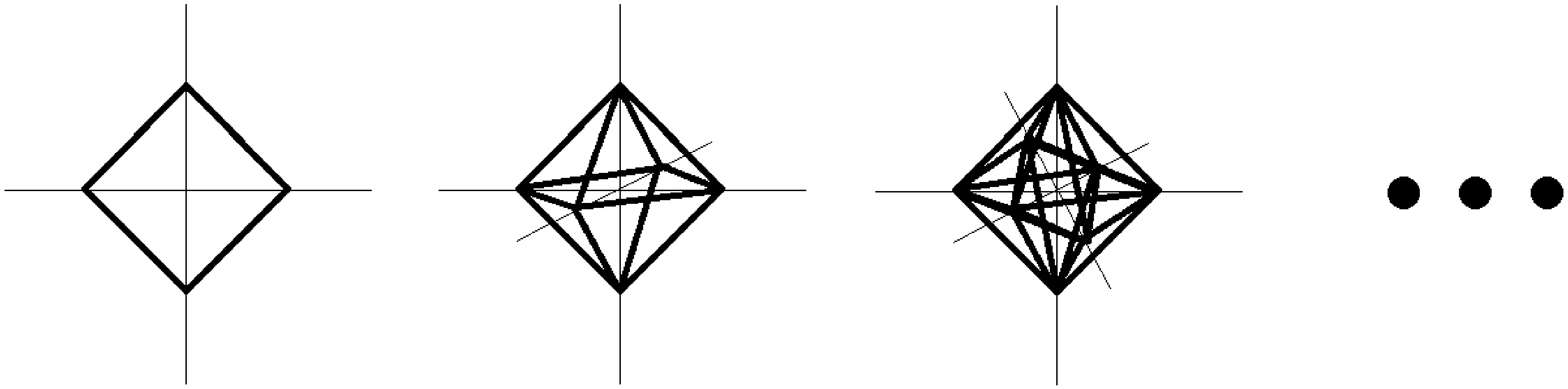
Higher Dimensional Tetrahedron Analogues

The Tetrahedral Family is n+1 points arranged at equal distance to each other in n-D space. Some inductive geometry can prove the center of the figure will always remain closer to all the vertexes than the vertexes are to each other, such when the center point is pushed out into the next highest dimension it can always move out to a point so it is the same distance from every vertex as the vertexes are apart. This can go on forever, so there is a regular tetrahedron analogue, always with n+1 sides and n+1 vertexes, in every dimension.



Higher Dimensional Octahedron Analogues

The cube must always have a dual in every dimension, which will always be an octahedron analogue. These can be easily conceived by taking all the coordinate axes of that dimension and putting vertexes at -1 and +1 along each of these axes, which gives the vertexes of the dual of a cube bounded by cutting planes at these same values.



All the Regular Polytopes

2D: Polygons – 3 or more sides, up to infinity.

3D: 5 Platonic Solids: T3 C3 O3 I3 D3

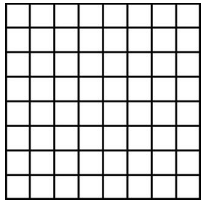
4D: 6 Regular Hypersolids: T4 C4 O4 H4 I4 D4

5D & above: 3 Regular n-solids: Tn Cn On

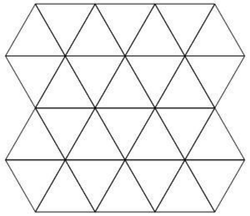
Apparently geometry can't accommodate diverse regularity in higher dimensions! Does this have some deeper meaning or bearing on why our existence has 3 macroscopic spatial dimensions (4 counting time?)

2D Regular Lattices

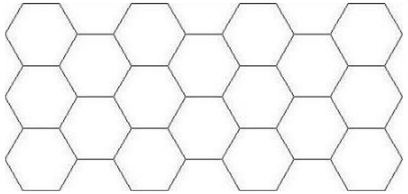
2D Space can be tiled by a C2 Square lattice



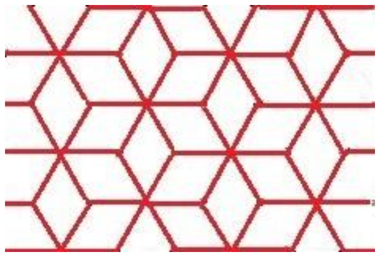
A T2 Equilateral Triangle lattice



A H2 Hexagonal Lattice

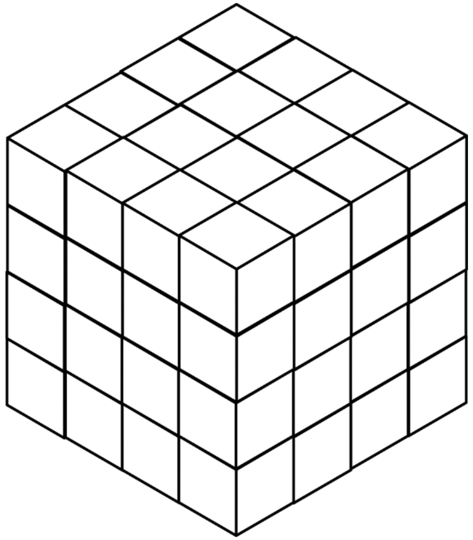


And a less than regular Diamond lattice
(Important for 4D)

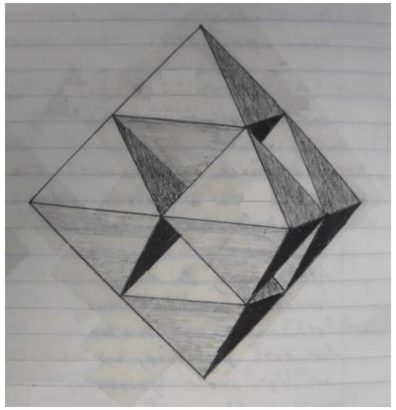


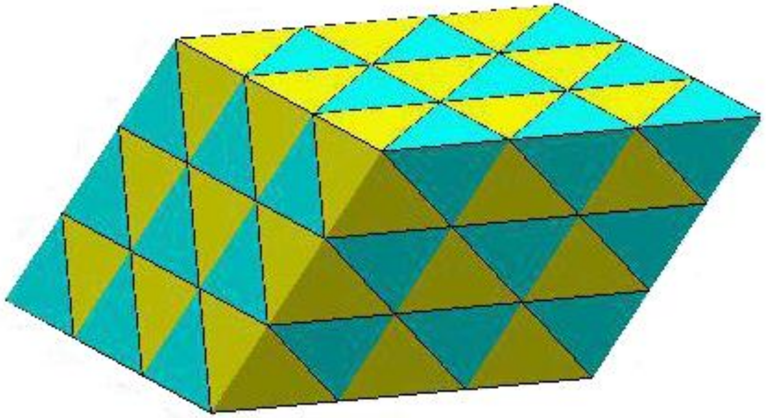
3D Regular Lattices

3D Space can be tiled by Cubes



Alternating Tetrahedrons and Octahedrons





4D Regular Lattices

C4 Cubes

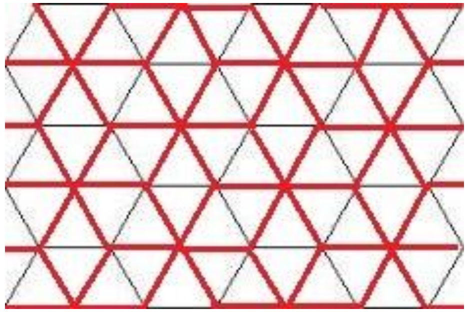
T4/O4 does NOT work

H4 Works much like H2 Hexagons in 2D

O4 also works!!!

O4 Lattice

While hard to illustrate, conceptually, the relation of O4 to H4 is like that of H2 to Diamonds in 2D, except the O4 is a regular polytope:



This can deduced from O4 Definition:

PLUS H4 has O3 sides at the proper distance from its center for its O3 sides to be "equators" of O4s all meeting at the center of the H4.

Regular Lattices

2D: Squares, Equilateral Triangles and Hexagons

3D: Cubes, Tetrahedron/Octahedron

4D: 4-Cubes, H4 and Octahedron analogue.

5D and above: n-Cubes ONLY!

Non-cubical patterns (that represent more spatially efficient ways to pack balls and provide stronger truss structures) die out if there are too many dimensions! Does this mean something?

APPLICATIONS

Highlight Uses of Higher-D Visualization & Geometry

Mobius

Complex Exponential Function

Hyperbolic and Complex Trigonometry Demystified

Fractals

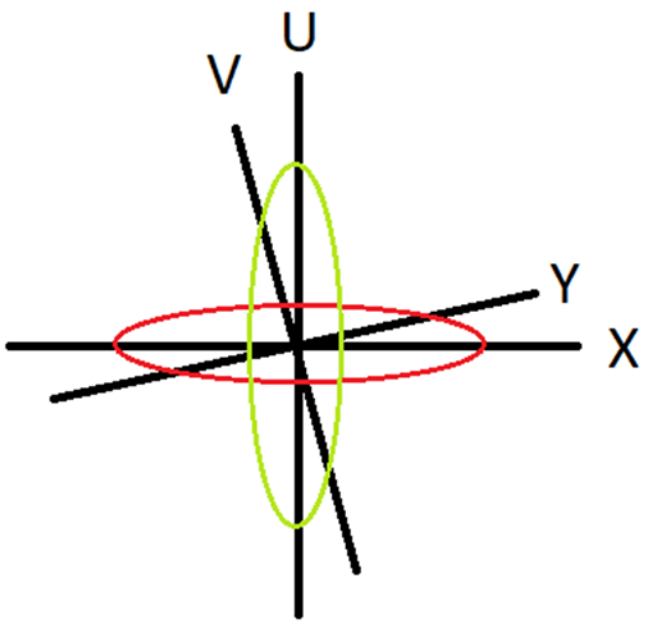
The Easy Solution

Differential Equations = Geometry

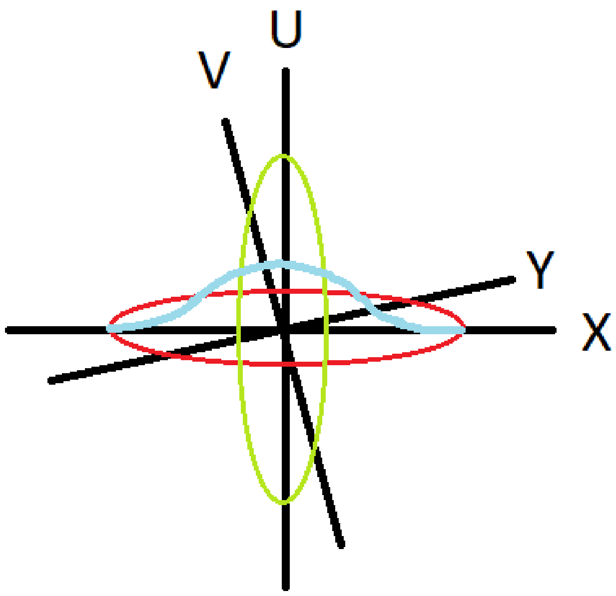
Complex Variables & Integration = More Pretty Pictures

Mobius Disk Construction

You can have 2 independent rotations in 4D space



Imagine rotating the blue bell curve shape this way.

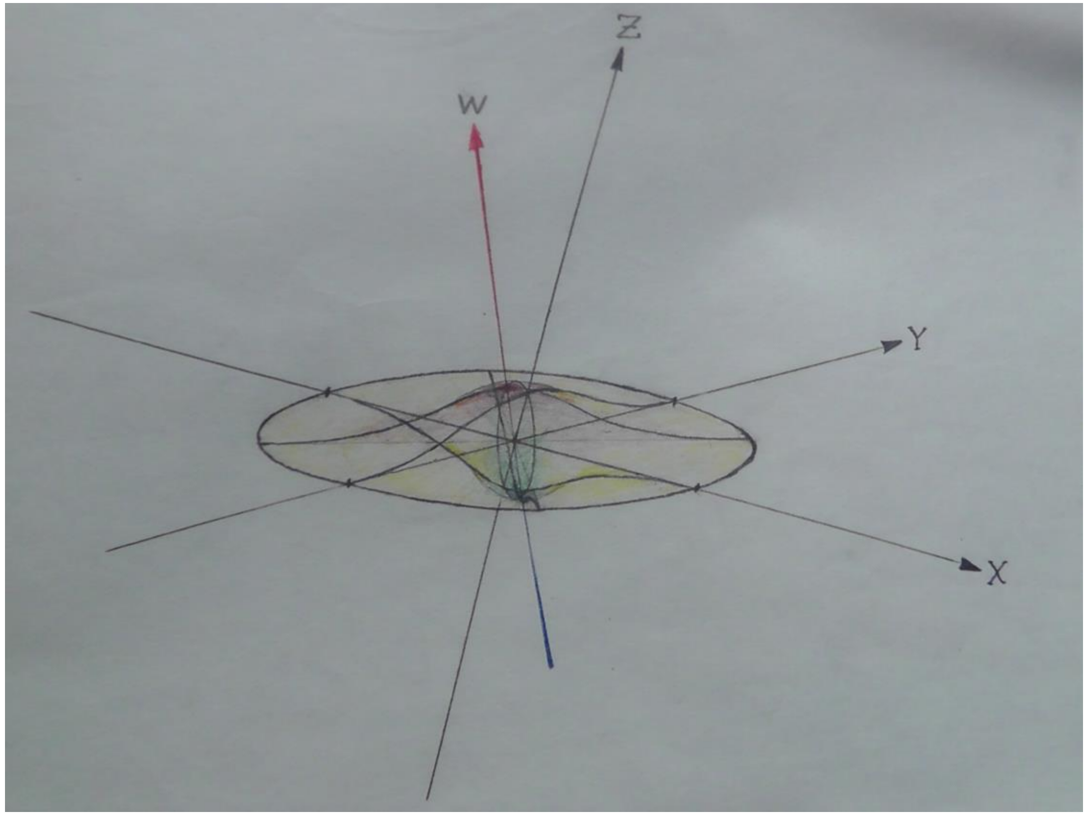


Mobius Disk

You get a Mobius Disk. Same as if stretch a Mobius Strip in 4D space such its single edge forms a circle.

Cross it and it flips you over!
Left <-> Right

Center (X,Y)=(0,0) of disk is NOT a point, but rather a circle in Z,W plane,

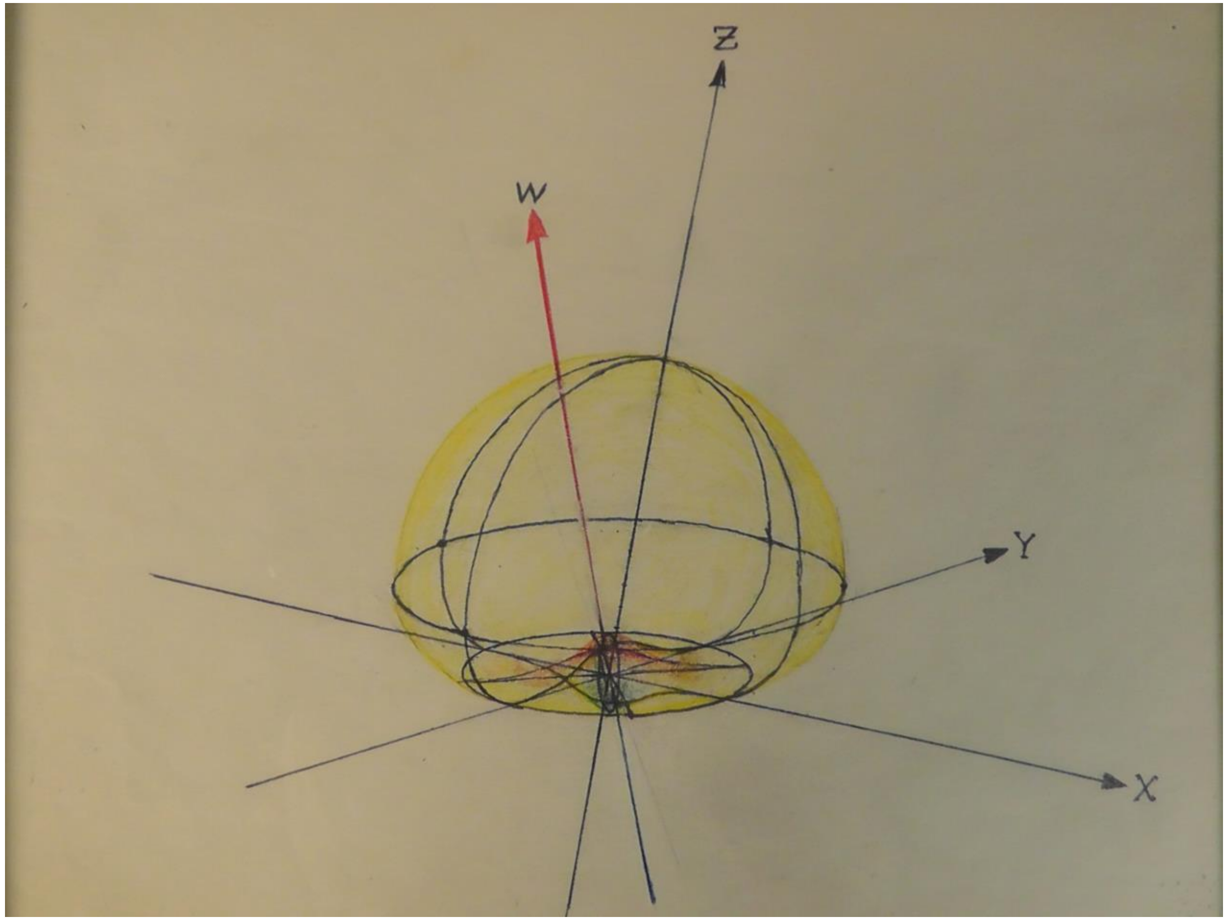


Mobius Sphere

Cut a hole in a sphere and sew on a Mobius Disk.

You get a 1 sided sphere!

However it is in 4D space, where a 2D surface can’t divide the 4D space to have an inside or outside.



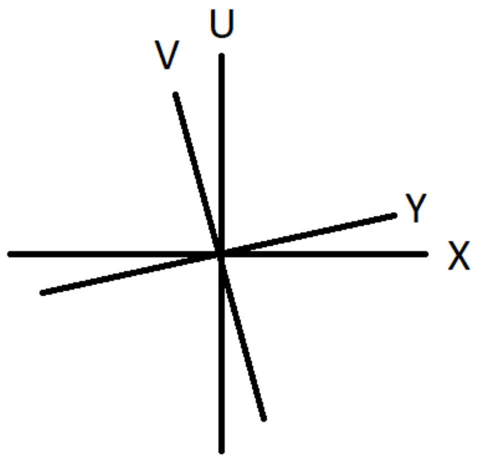
Complex Variable Functions/Calculus

<u>Question</u>: What is Square Root of -1?   <u>Answer</u>: ±i (Imaginary Number)

Complex numbers/variables are of form: z = x +iy.

F(x)=y (2D) becomes F(z)=w: F(x+iy) = u=iv (4D)

Z & W = 2 perpendicular planes intersecting only at 0.



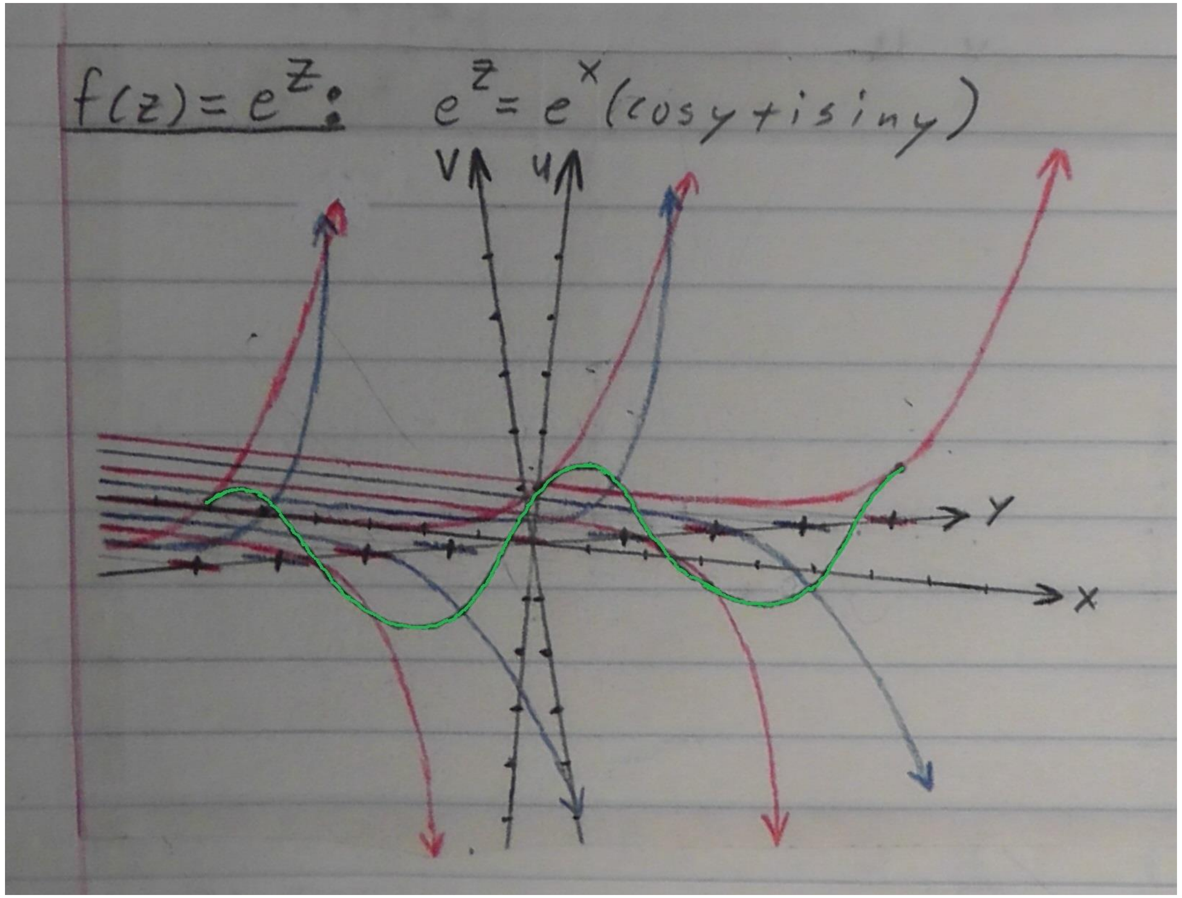
Complex Exponential Function

$F(z) = e^z \quad z=x+iy$

$= e^x[\cos(y) + i\sin(y)]$

Get red e^x in X,U real X,U plane.

$|z|=1$, ($e^{i\theta}$ green circle in U,V plane) is helix around Y (θ) axis.



Complex Exponential Function

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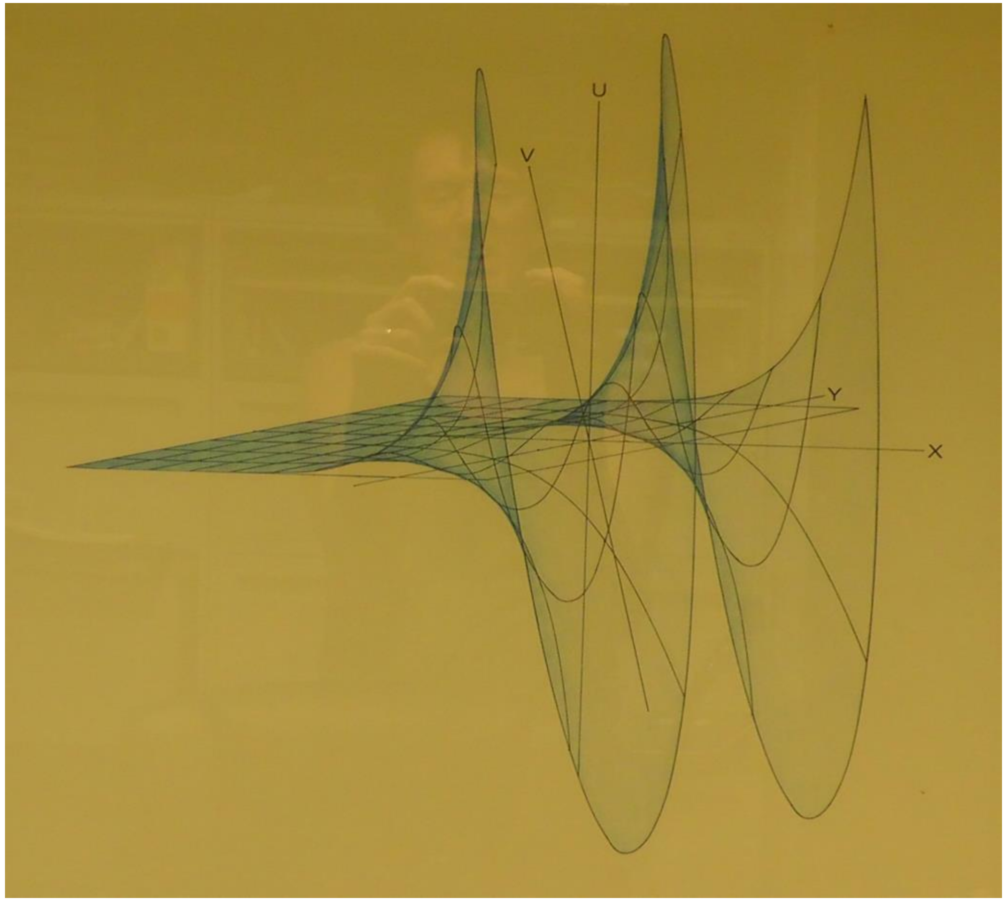
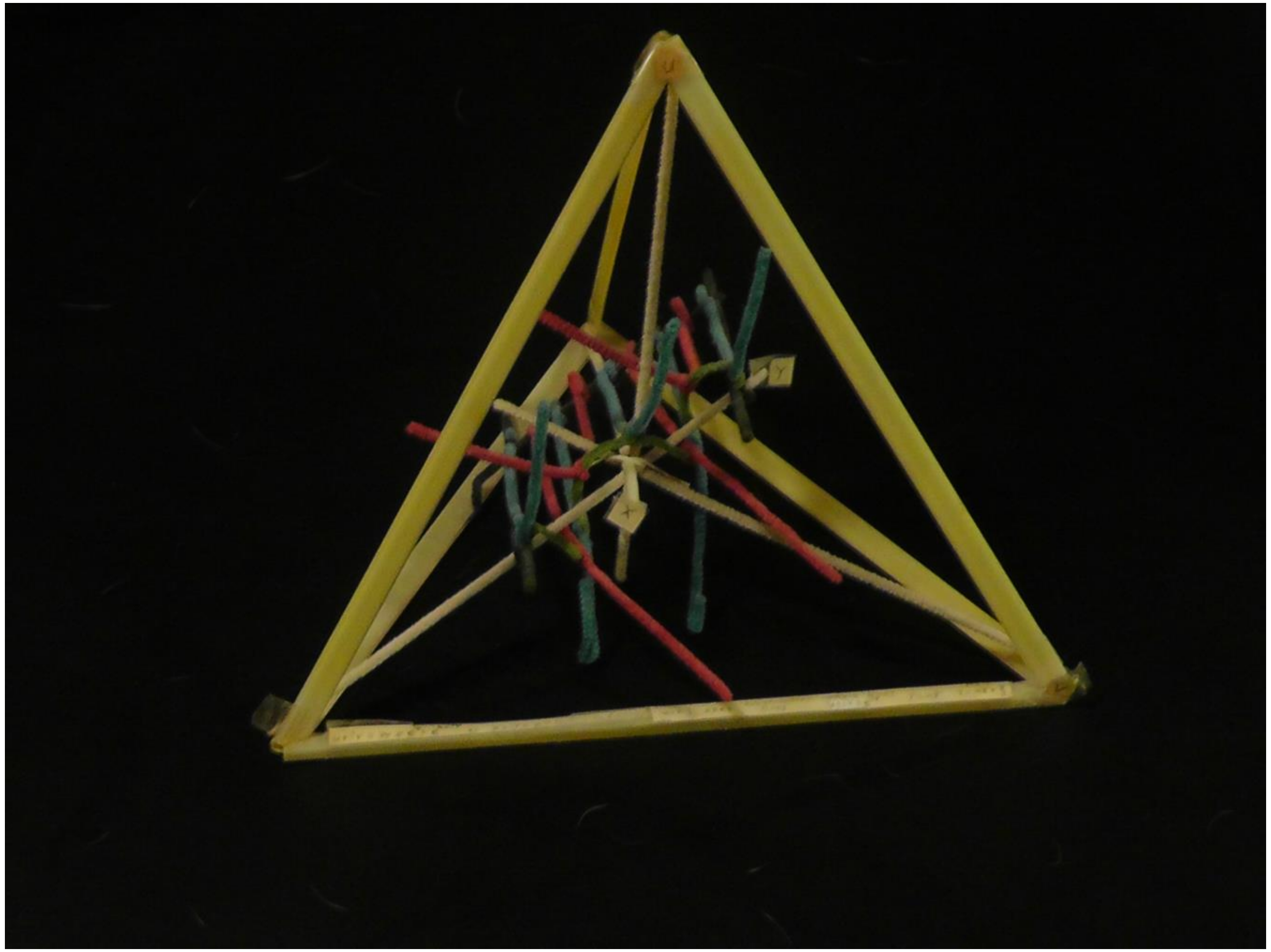


Surface = exponential corrugation that spirals without intersecting itself.

Complex Exponential Function

3D Model of it



F(z)=sin(z)

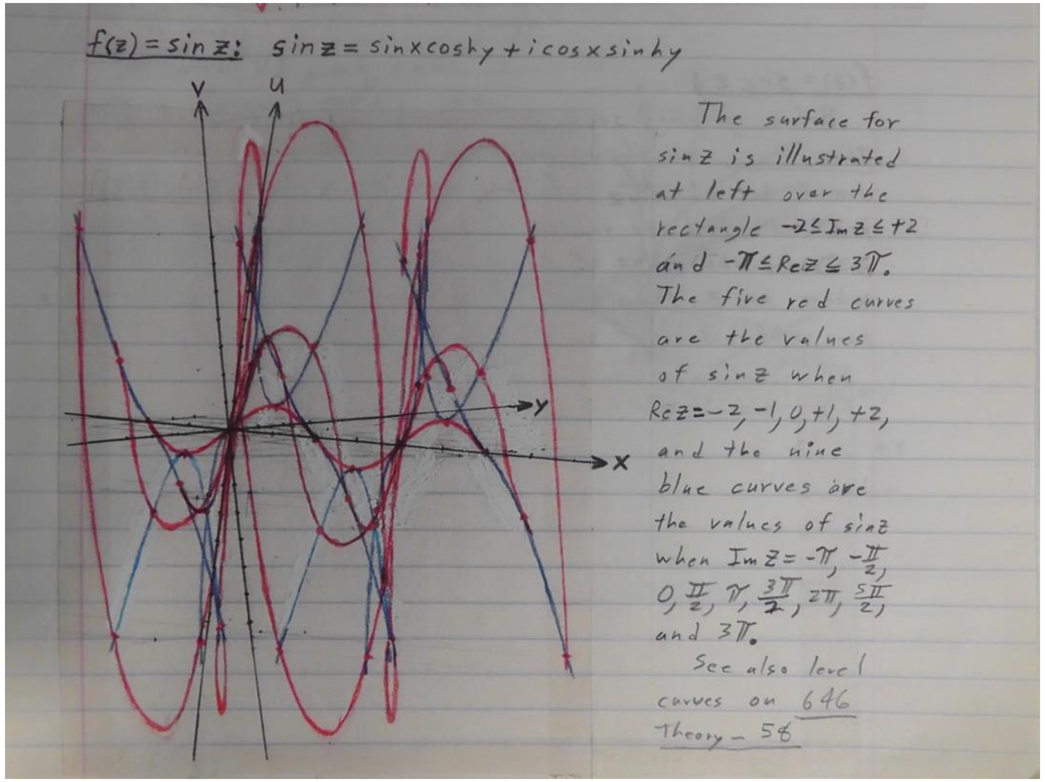
$\sinh(z) = [e^{z} - e^{-z}] / 2$

$\sin(z) = [\sinh(iz)] / i = [e^{iz} - e^{-iz}] / 2i$

Similarly $\cosh(z) = [e^{z} + e^{-z}] / 2$

$\cos(z) = [\cosh(iz)] = [e^{iz} + e^{-iz}] / 2$

cos(z) algebra meaning:
iz = z twisted 90°, -iz twisted -90°. Take the average (add and / 2). The resulting surface exponentially spiral corrugates on both sides of a y=0 sine/cosine wave "central spine" created when the rotated $e^{i\theta}$ parts destructively cancel the imaginary part to add to leave only a real sine or cosine wave when y=0 (Z is real).



This is how trig & hyperbolic trig can have similar algebra & calculus!

F(z)=1/z

Red F(x)=u (real hyperbola)

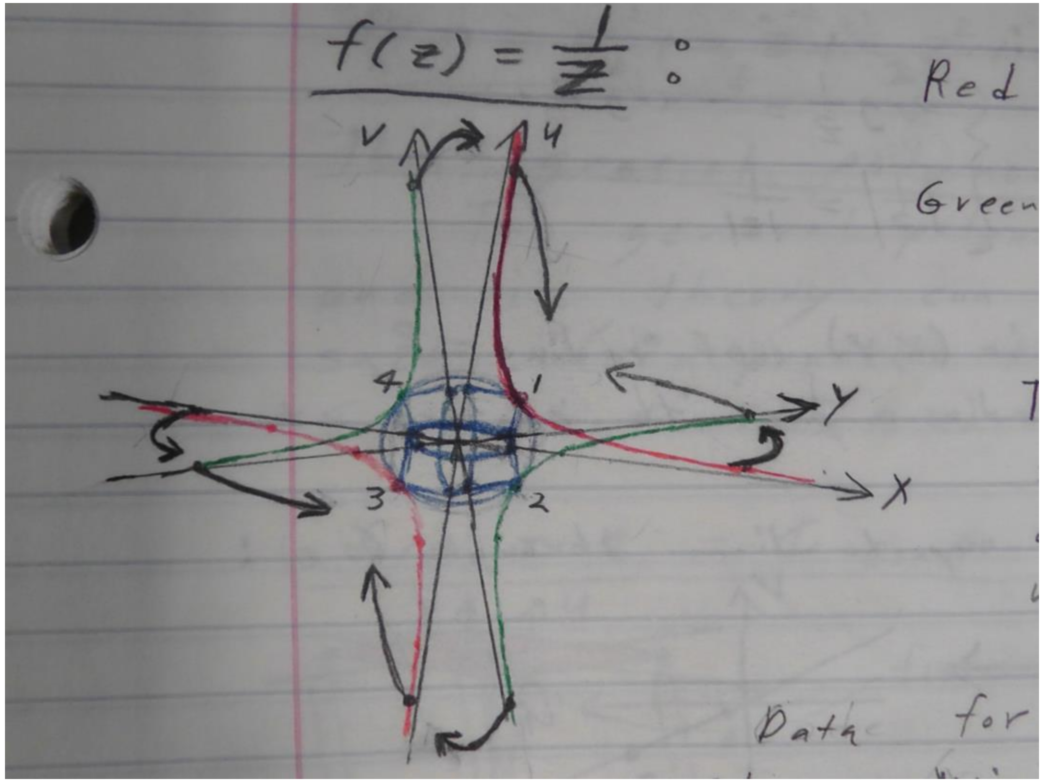
Green F(-iy)=v (Imaginary hyperbola)

Points 1,2,3,4 & blue map unit circle in x,y to a 4D curve showing how 2D surface is made be spinning hyperbola legs:

Rotation 1: in Domain (X,Y) Plane

Rotation 2: in Range (U,V) Plane

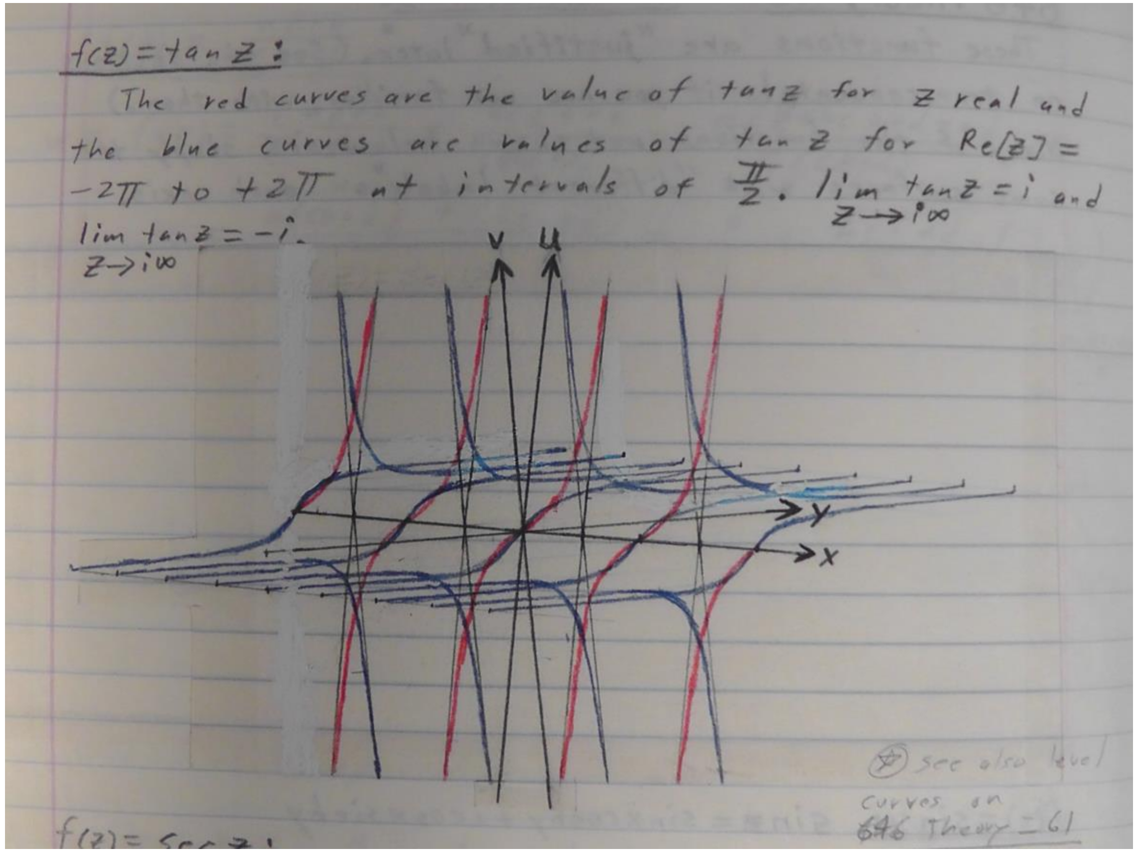
Each plane = axis for other plane!



F(z)=tan(z)

Tan(z) is another periodic function.

Notice it has singularities that spin in U,V like a 1/Z singularity does.



Complex Unit Circle and Trigonometry

For what it is worth, you can develop a complete complex extension of the notions of angle and trig functions!

GEOMETRIC MODEL FOR COMPLEX TRIGONOMETRY

Complex Unit Circle: $Z^2 + W^2 = (X+iY)^2 + (U + iV)^2 = 1$

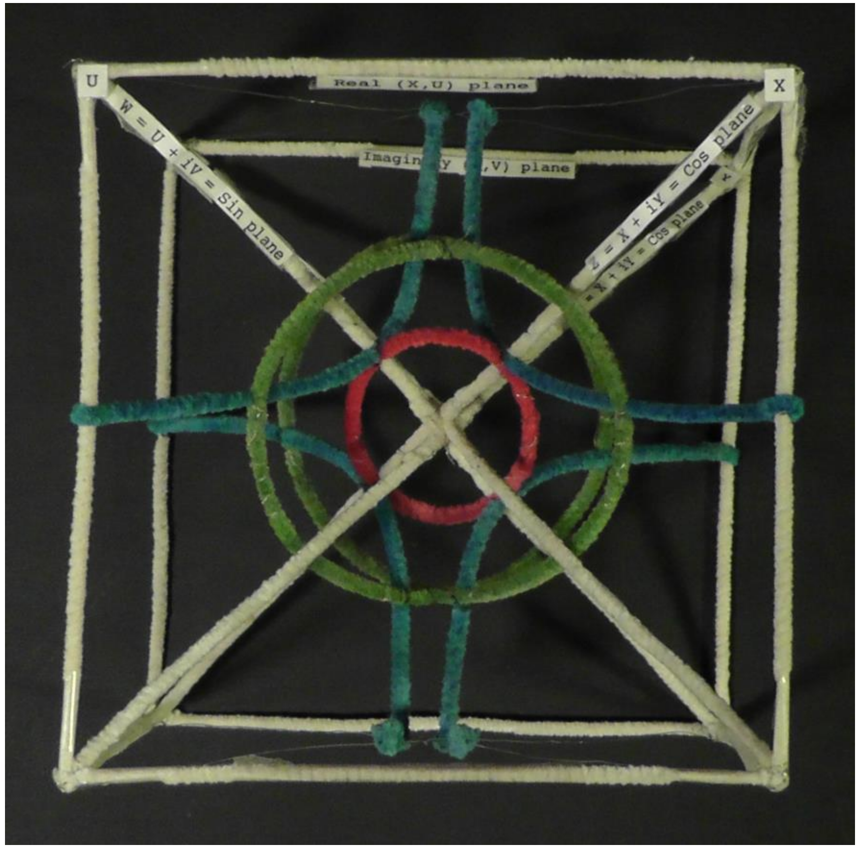
Angle: $\theta = \alpha + i\beta$

($\beta = 0$) Real Unit Circle $X^2 + U^2 = 1$
(Round looking down from top)

$\beta \approx \pm 1$ Circles

$\alpha = 0, \pi/2, \pi$ & $3\pi/2$ Hyperbolas

$\sin\theta$ = W component = U component + iV component = $\sin(\alpha)\cosh(\beta) + i\cos(\alpha)\sinh(\beta)$
$\cos\theta$ = Z component = X component + iY component = $\cos(\alpha)\cosh(\beta) + i\sin(\alpha)\sinh(\beta)$



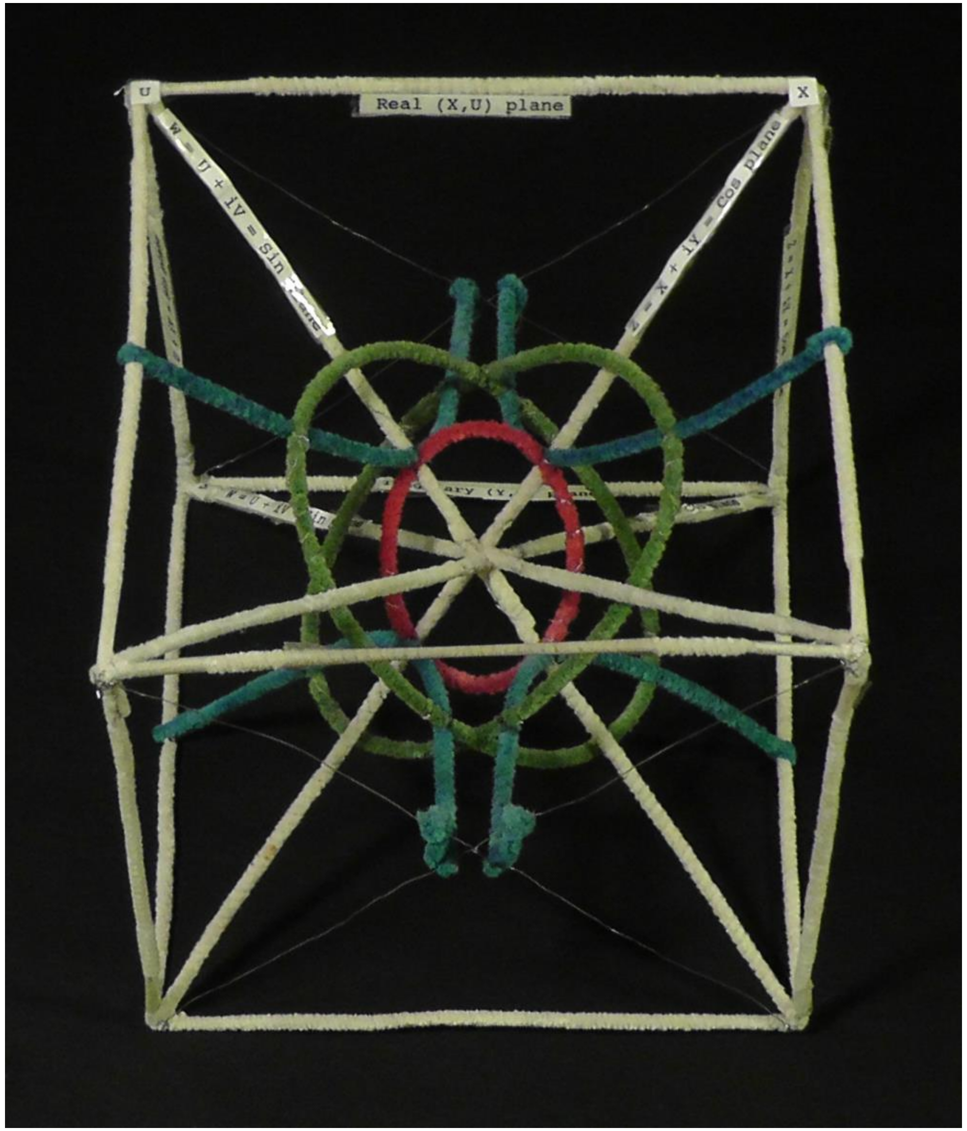
Complex Unit Circle

Hyperbola (Blue) spins as it is spun around the red unit circle to create a surface that flips over as it goes around but does not have to pass through itself.

Angle $\theta=\alpha+i\beta$:

α is measured around red circle.

β is measured on a hyperbola.



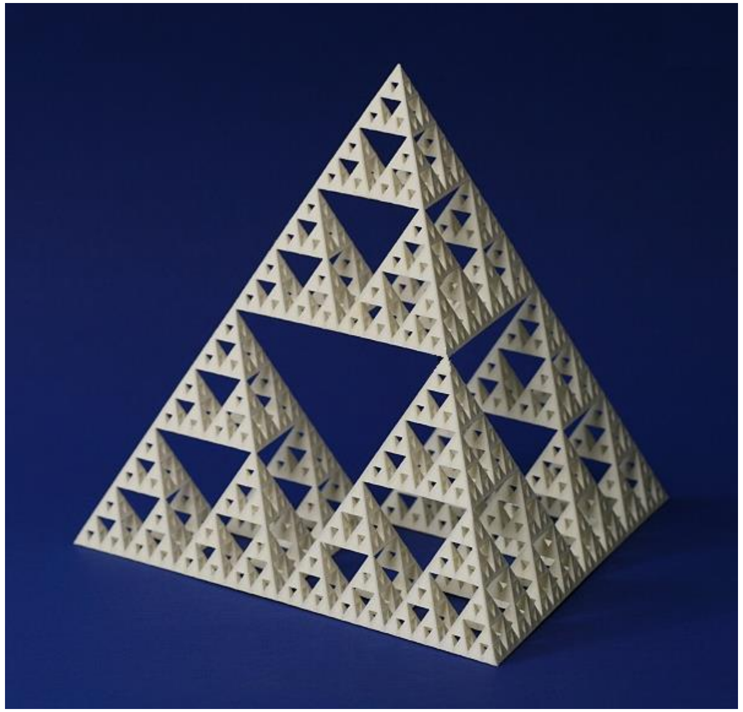
Further Work

I went on to research on 4D squig path fractals based on 4D lattice structures as part of my undergraduate math degree.

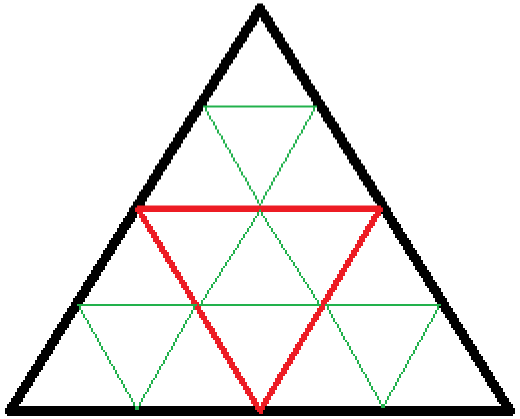
Higher dimensional visualization techniques allow things like linear algebra, differential equations and complex variable calculus to be made into geometric visualization problems to more easily see and understand theorems and solve practical problems by visualizing the answer, writing the solution equation and solving for a few constants to get answers. It is also helpful for studying things like relativity, and even more qualitative topics like modeling emotions with discontinuous changes using catastrophe theory mathematics.

Fractals

T3 Tetrahedron – Remove octahedral centers.

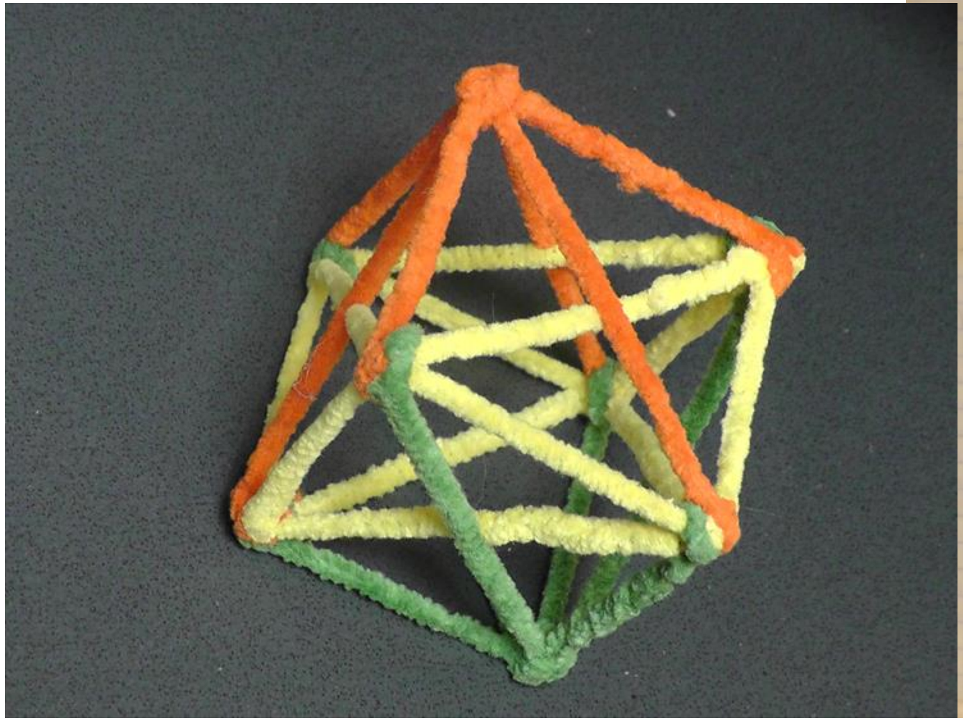


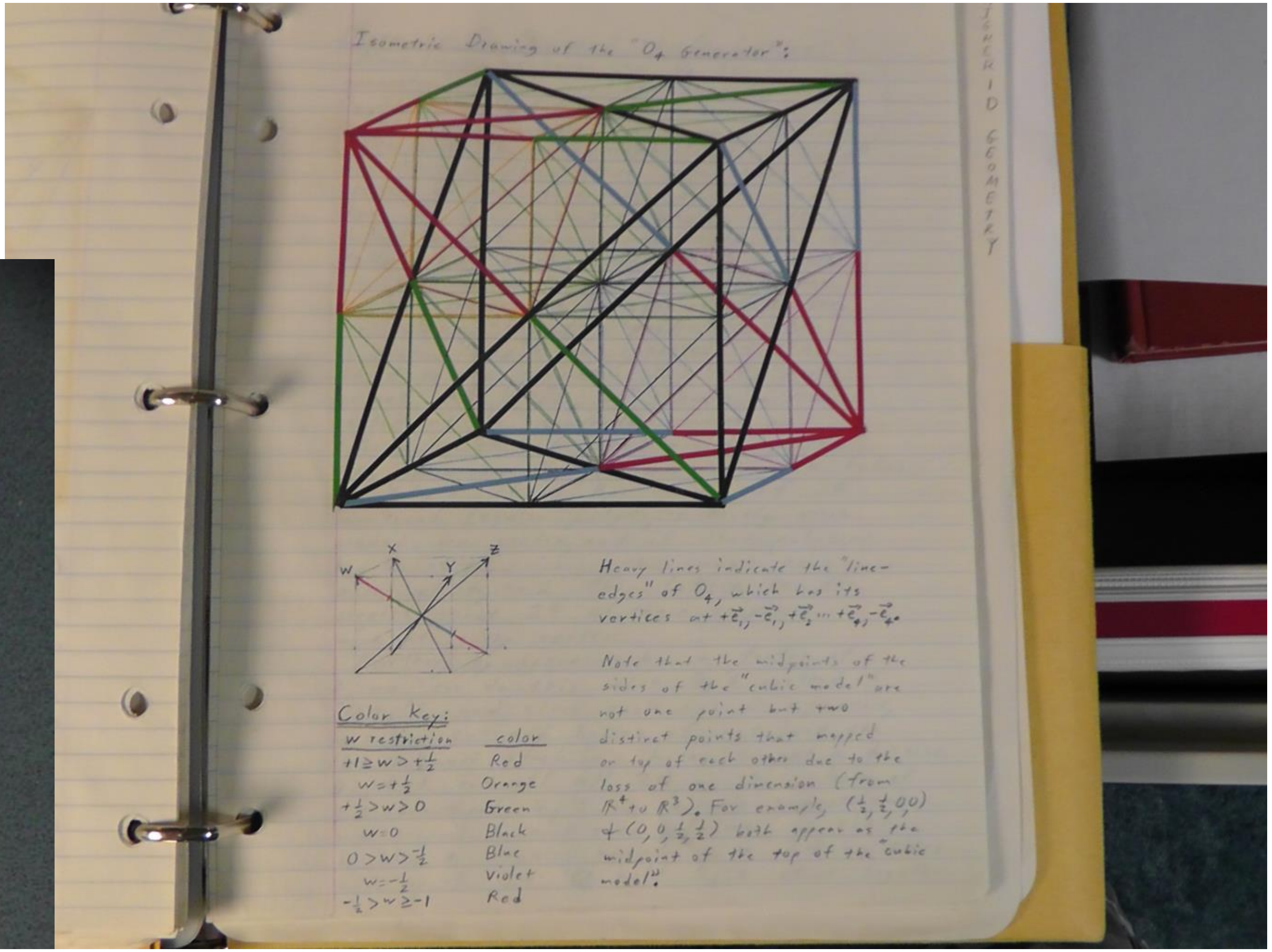
Plane view of each of 4 faces shows SAME 2D area as remove octahedral centers (after rotating the red back triangle). In the Limit you get a 2D area object.



A Hyperspace (4D) Filling Solid (3D) Fractal

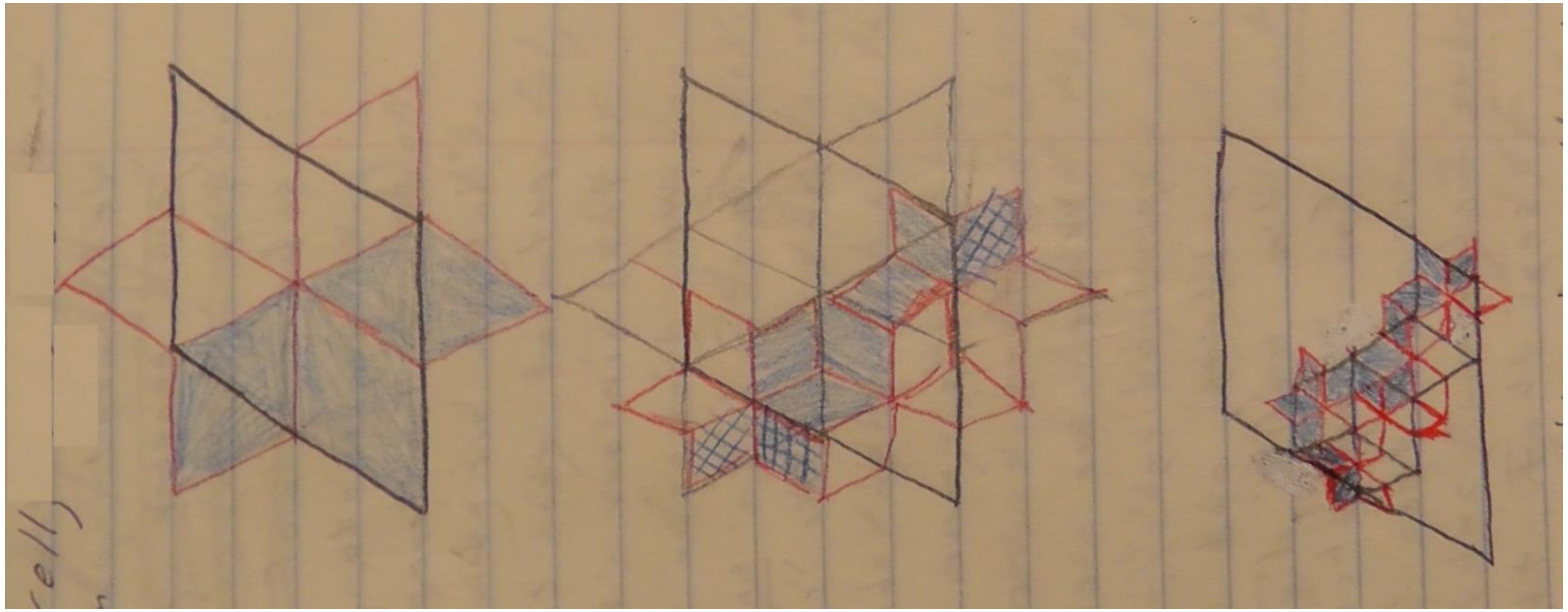
O4 (Isometric View) (3D for 4D)
8 half sized O3 fill Iso view cube.
Constant 3D Volume as split up.





Squigg Fractals

Concept: Divide up shape into smaller versions of same shape till you get an infinitely thin pathway.



I proved this works with O4 lattice.

The Easy Solution: Visualize the Answer

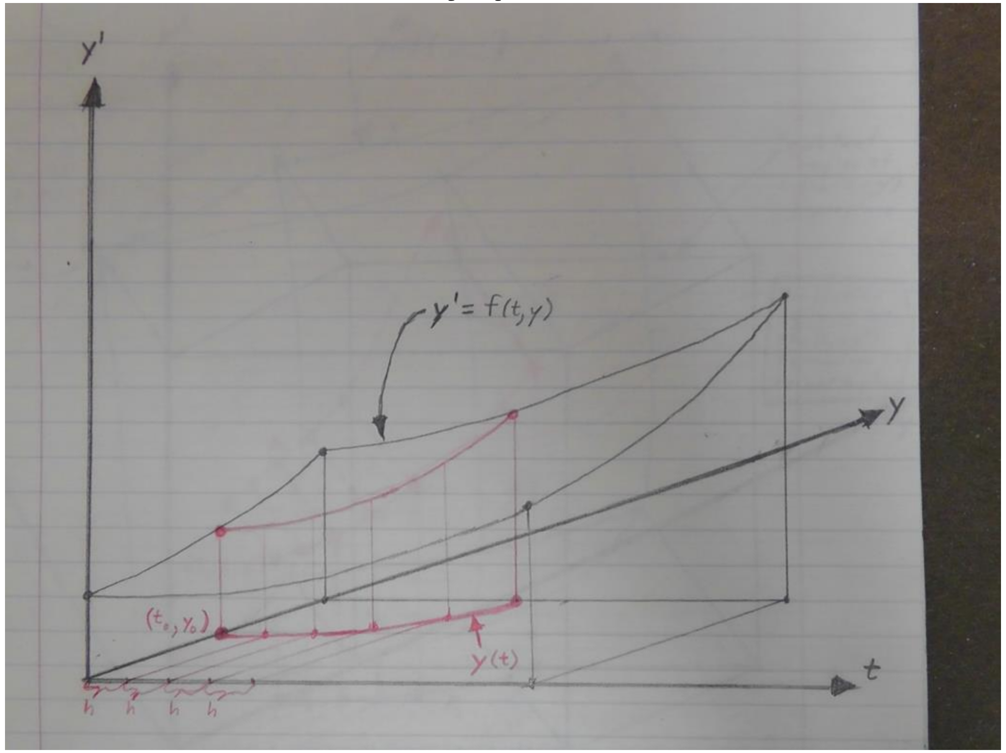
Sometimes you can take a problem like an integral or differential equation with boundary conditions and visualize it spatially to see what the answer ought to look like.

Then just write an equation for the answer and maybe play with the constants to dial in the solution and verify it works.

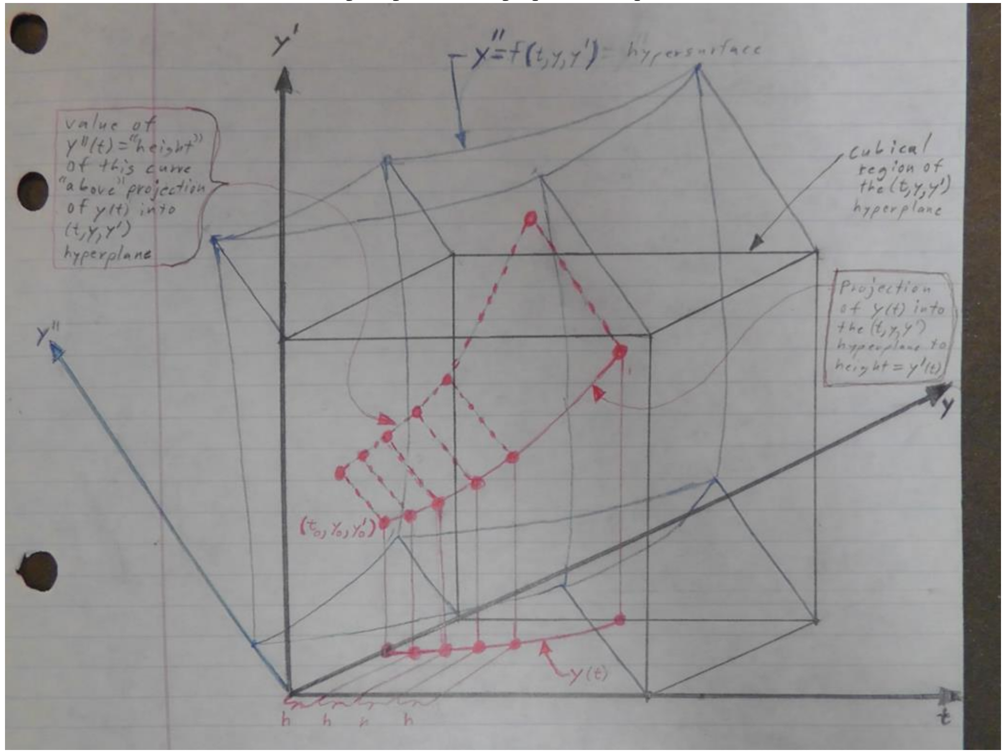
<u>Just One Complication</u>: It is only easy if you can visualize it.

Differential Equations (As Geometry)

1st Order $y'=f(t,y)$ Surface above t,y plane

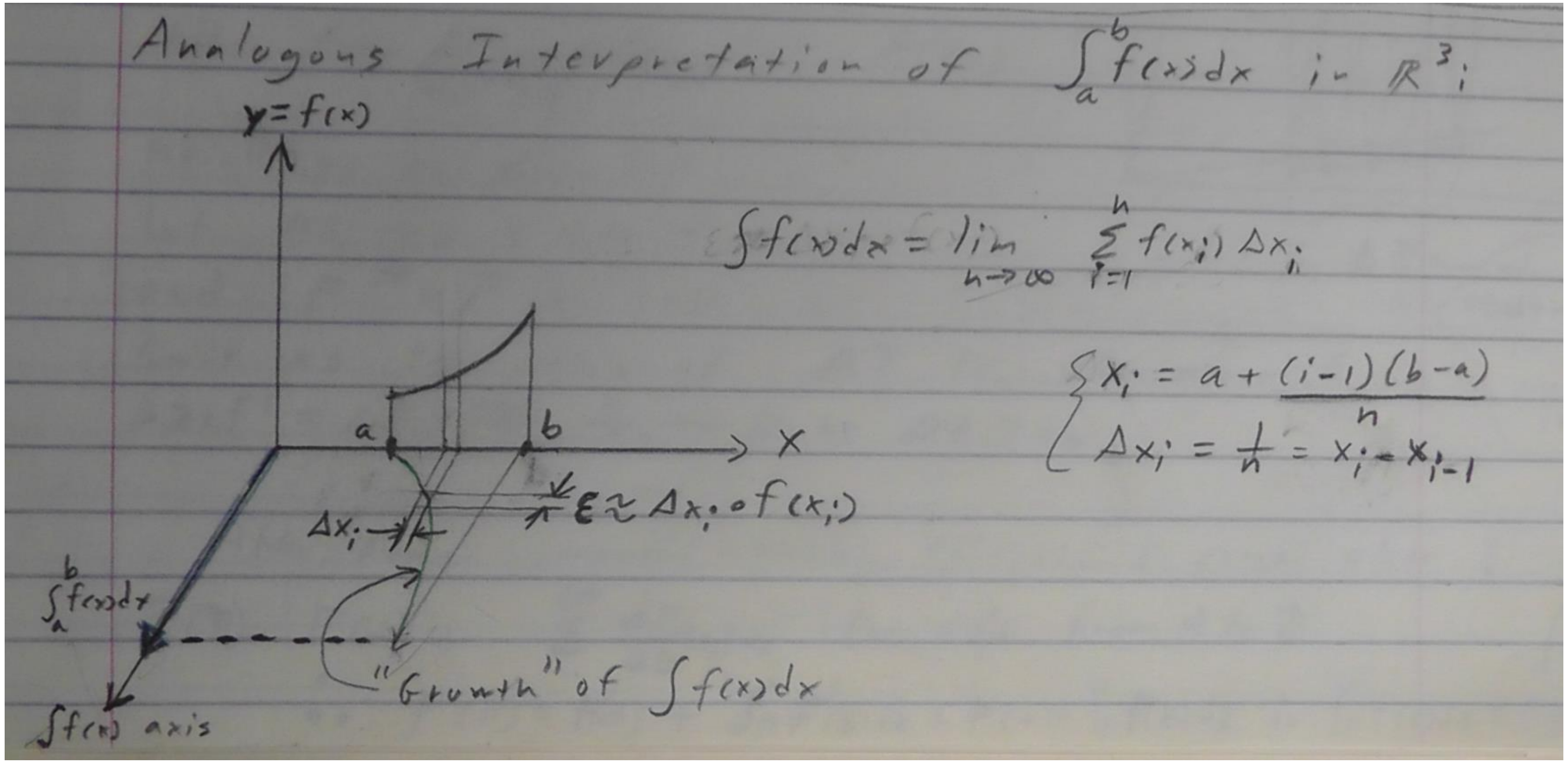


2nd Order $y''=f(t,y,y')$ Hypersurface above t,y,y' hyperplane



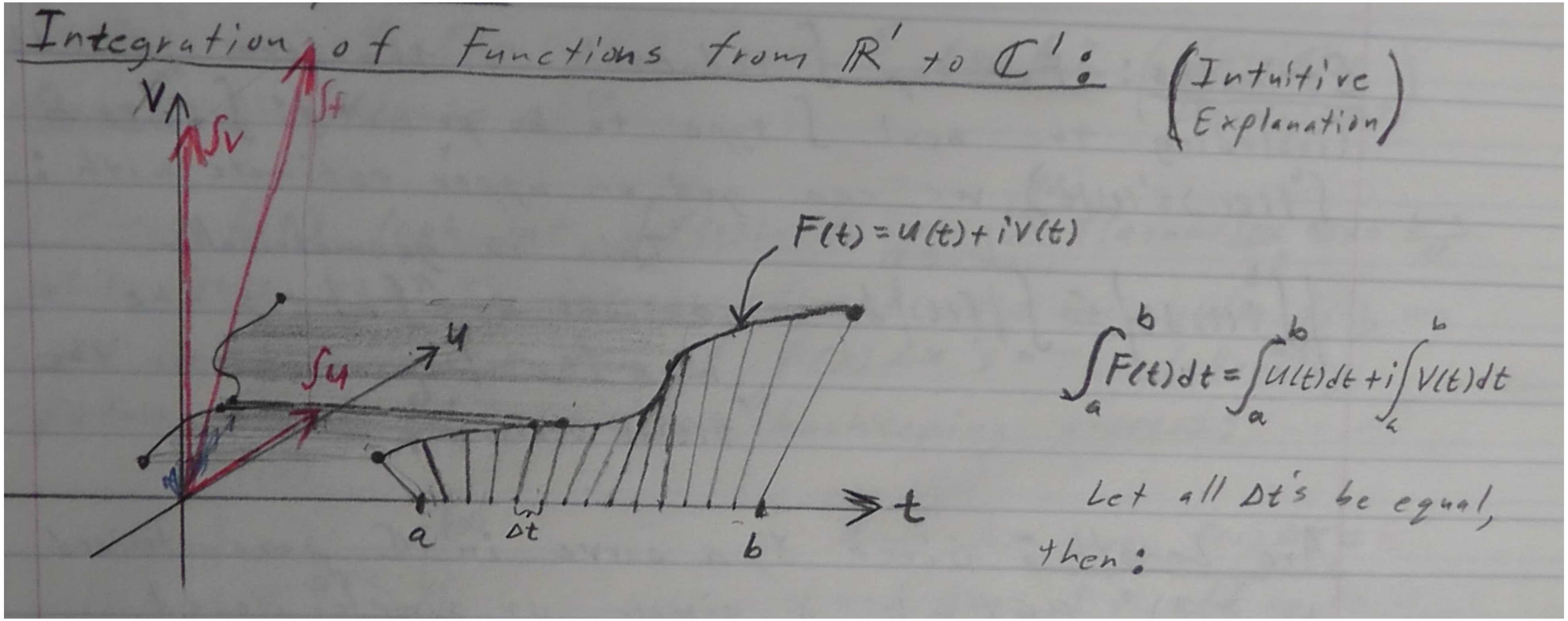
Integration as Summation

Real Variable



Real Parameter Complex-Valued Integration

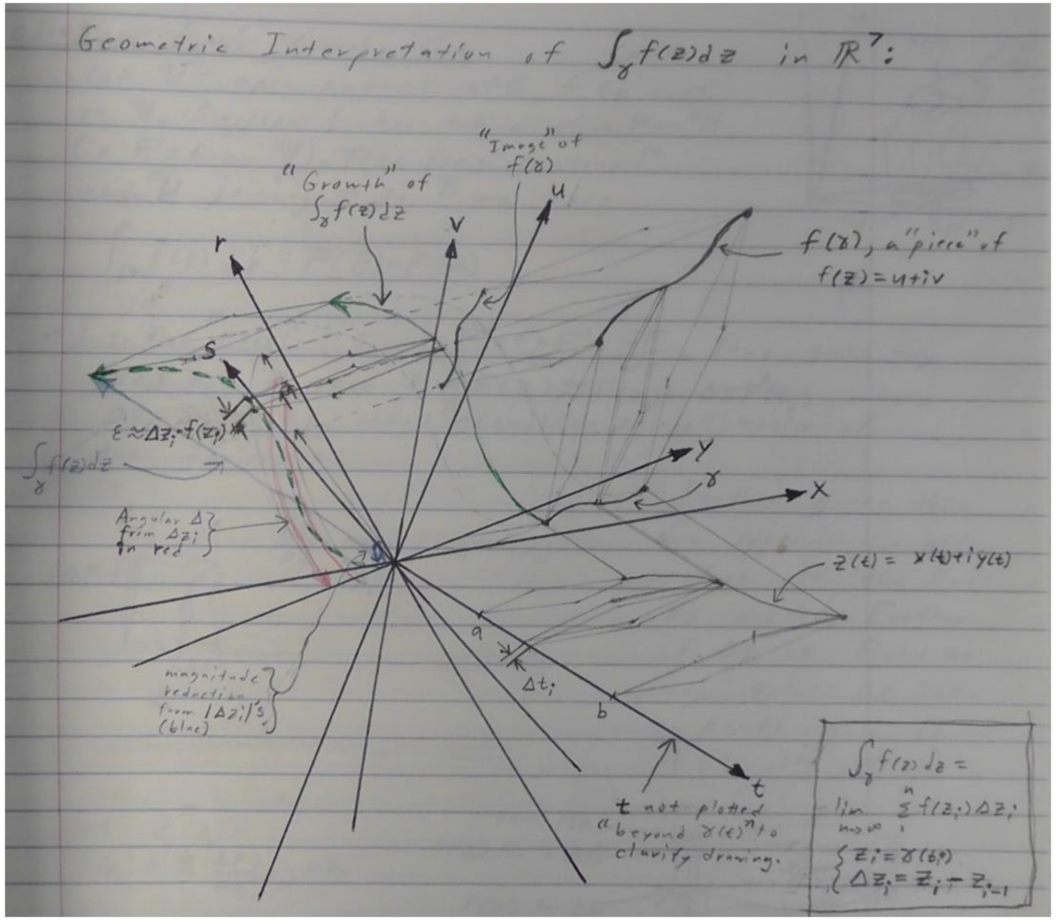
Integrate F(t)=u+iv



Complex Contour Integration

Integrate F(x+iy)=u+iv
Along Path
Parameterized by t

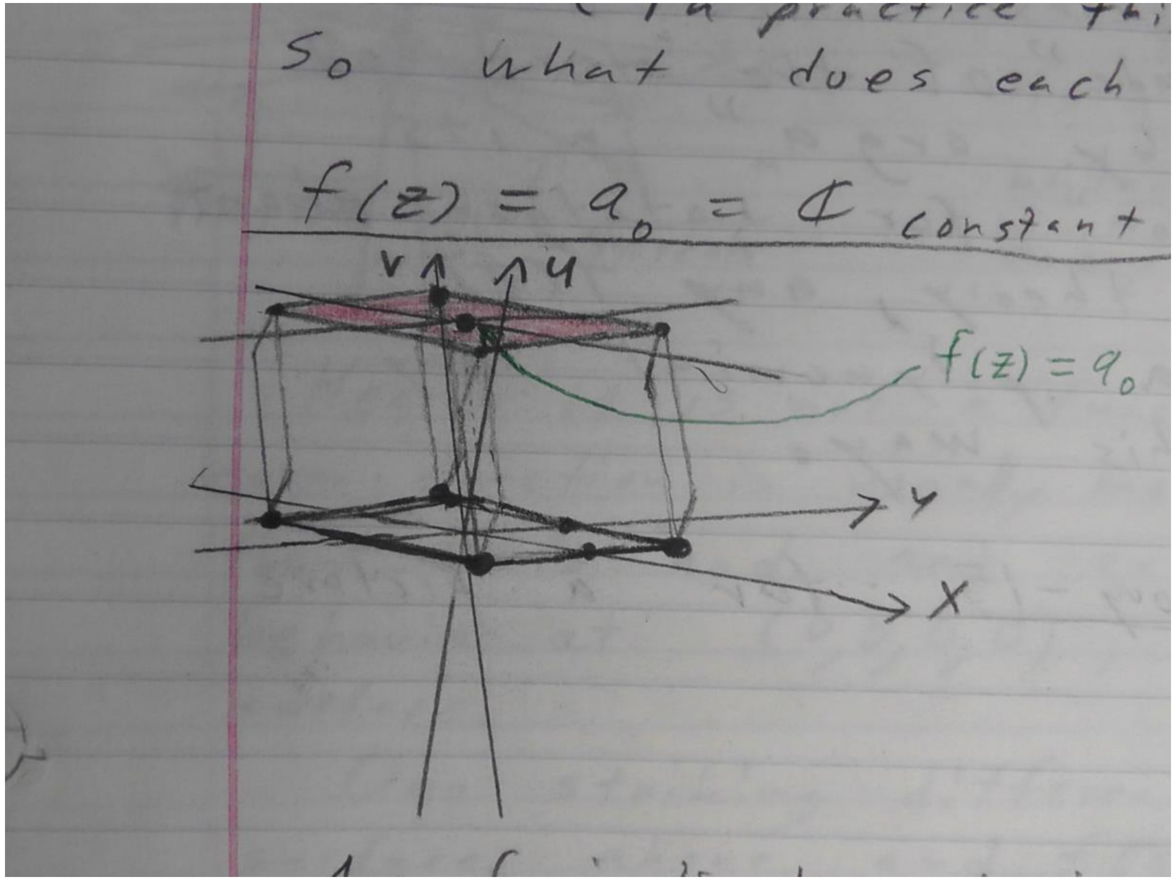
Is a 7 dimensional geometric process
embodied by algebraic calculus!



F(z)=Constant

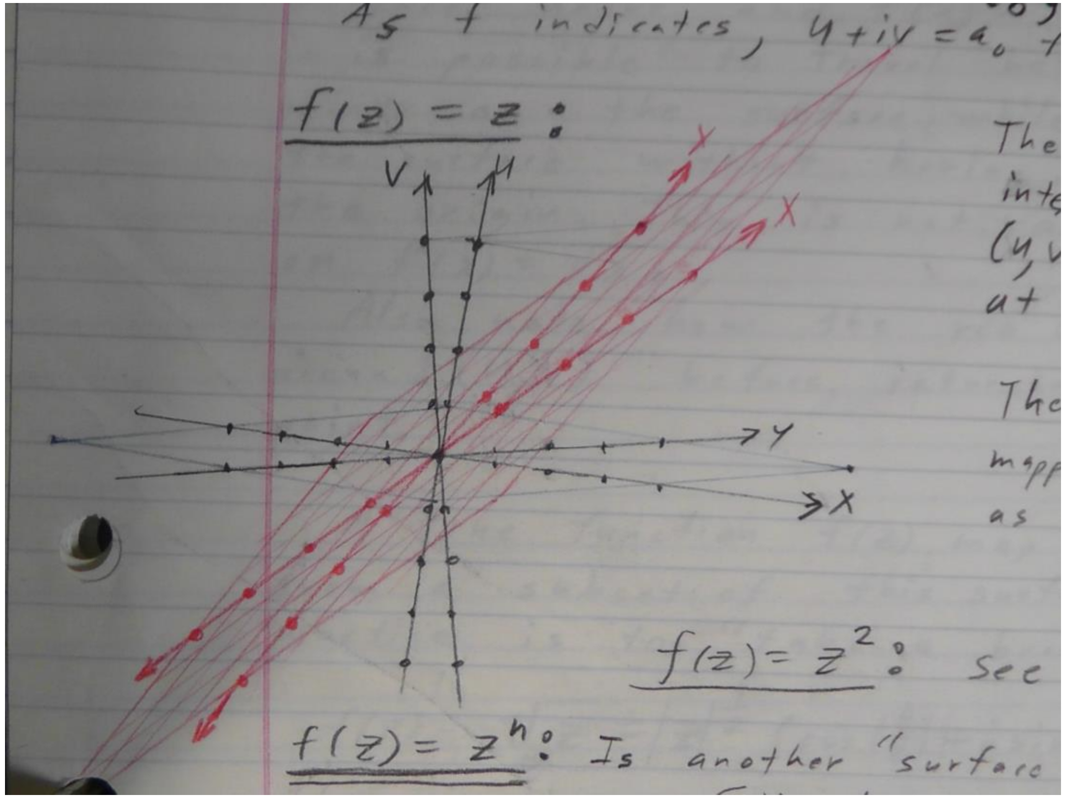
Looks like this:

Just a plane offset from x,y plane by a complex constant (point in u,v).



F(z)=z

A 2D plane in 4D space.

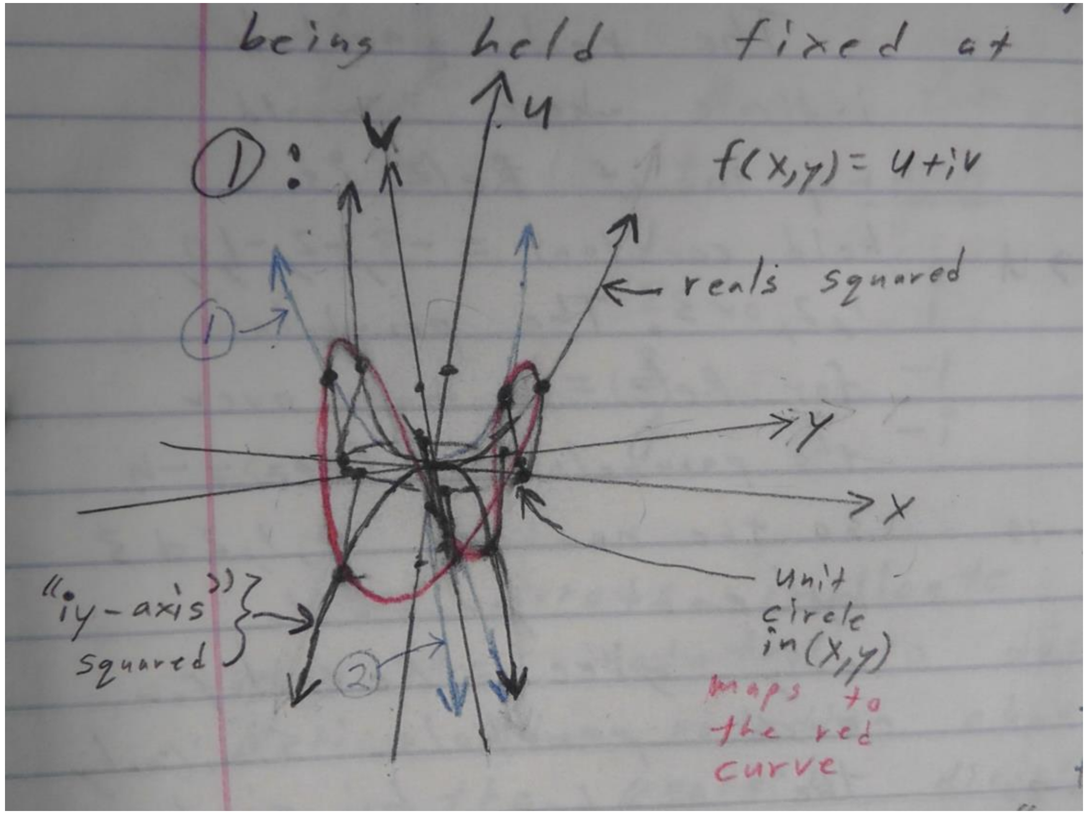


$F(z)=z^2$

"Reals squared" +u parabola in X,U plane.

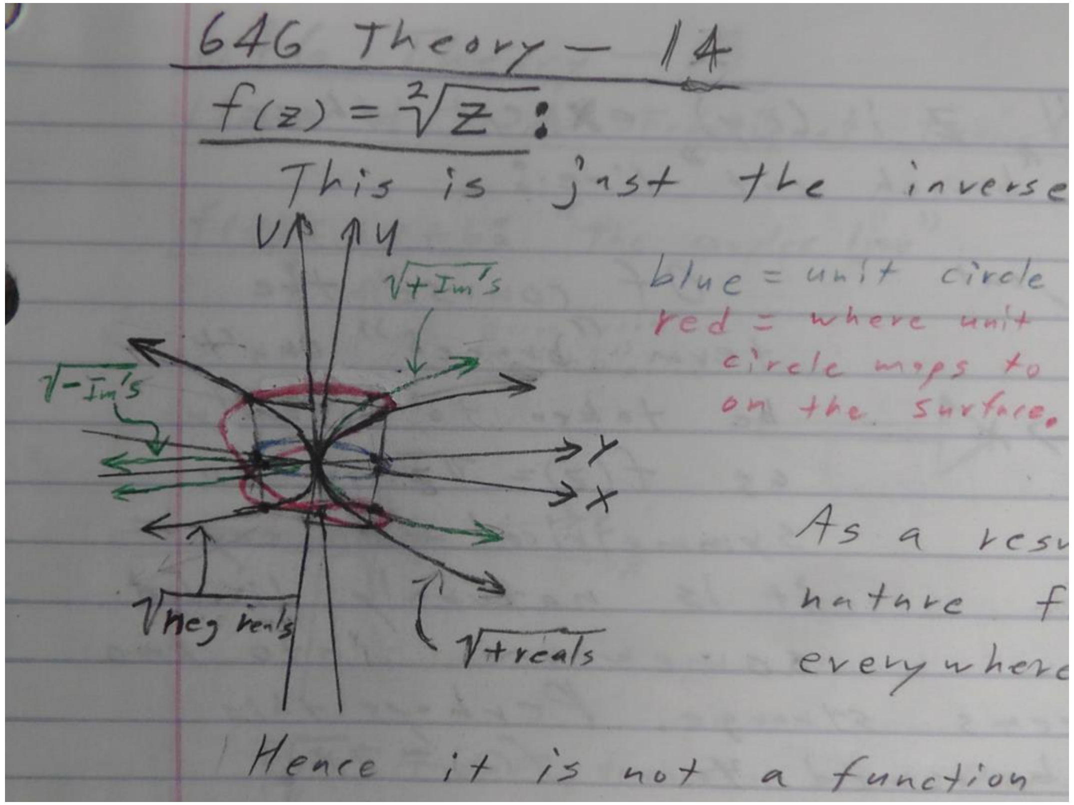
-u parabola for iy^2

For |z|=1 unit circle get red Curve showing how 2D surface "spins around twice" (=2 exponent) as go once around unit circle.



F(z)=Square Root (z)

Same shape flipped around such each x+iy maps to 2 root solutions (because 2nd root), (except 0 just maps to 0.)

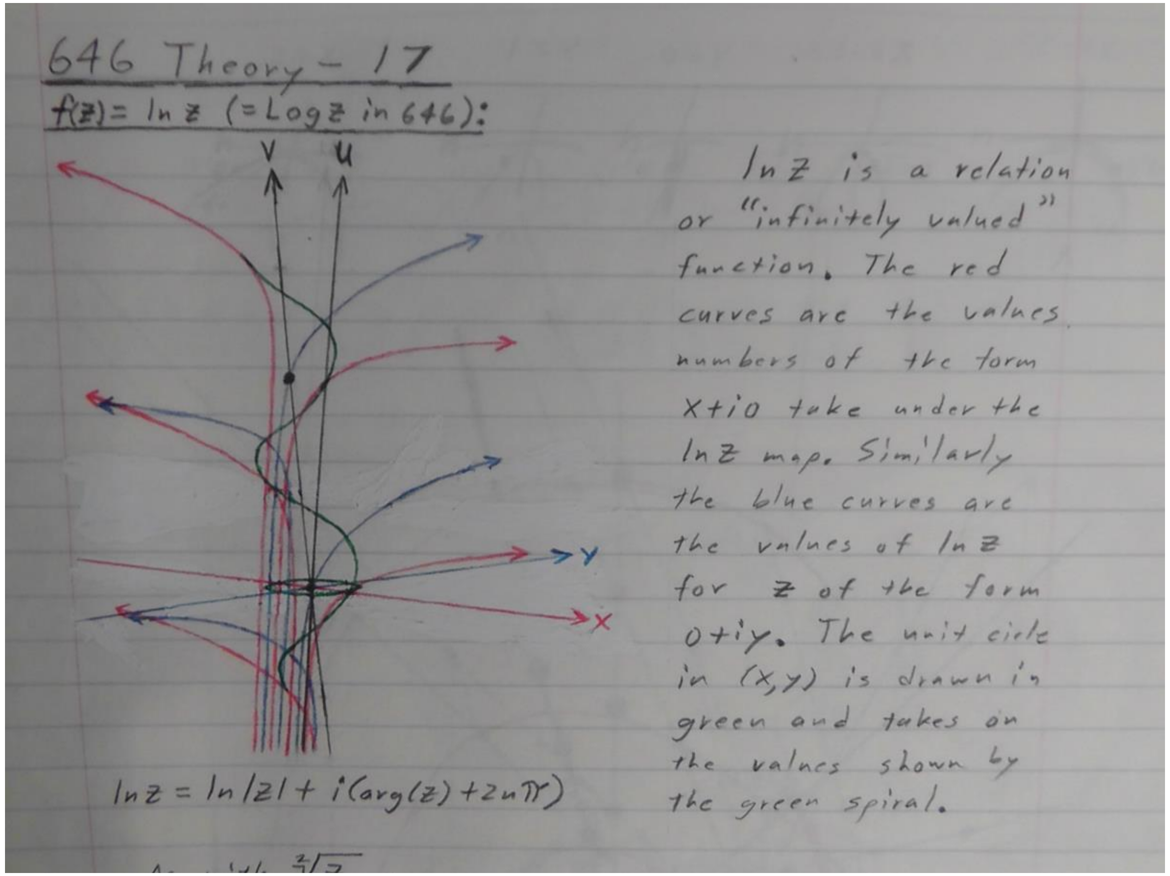


F(z)=Log(z)

Just flips exp axes around.

Get relation, because Log(Z) has many solutions intervals of 2π along V axis apart.

Surface is like a 4D version of a seashell spiral.



Questions?

You all get an “A” for “Attendance”

Be sure to exit the mobius sphere before returning to reality!

and

Don’t take any 4D matter out of this safe imaginary space!